

Getting Lost in the Wealth of Classifier Ensembles?

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The Leverhulme Trust

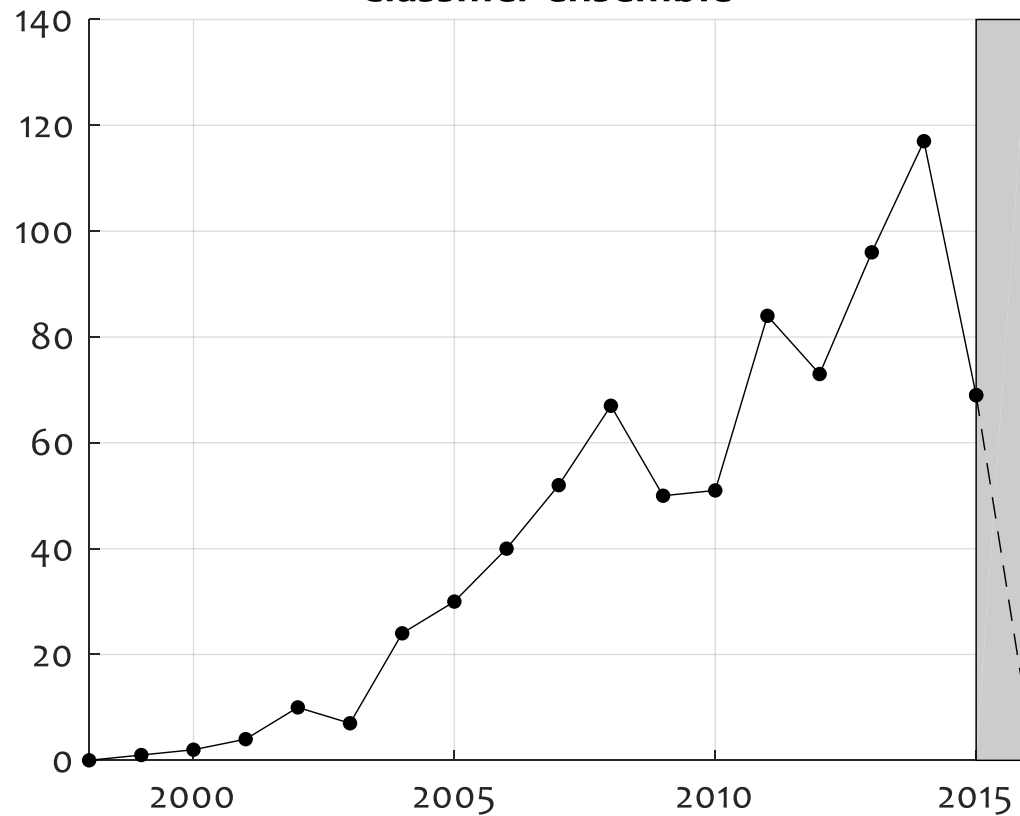
Supported by Project RPG-2015-188 Sponsored by the Leverhulme trust



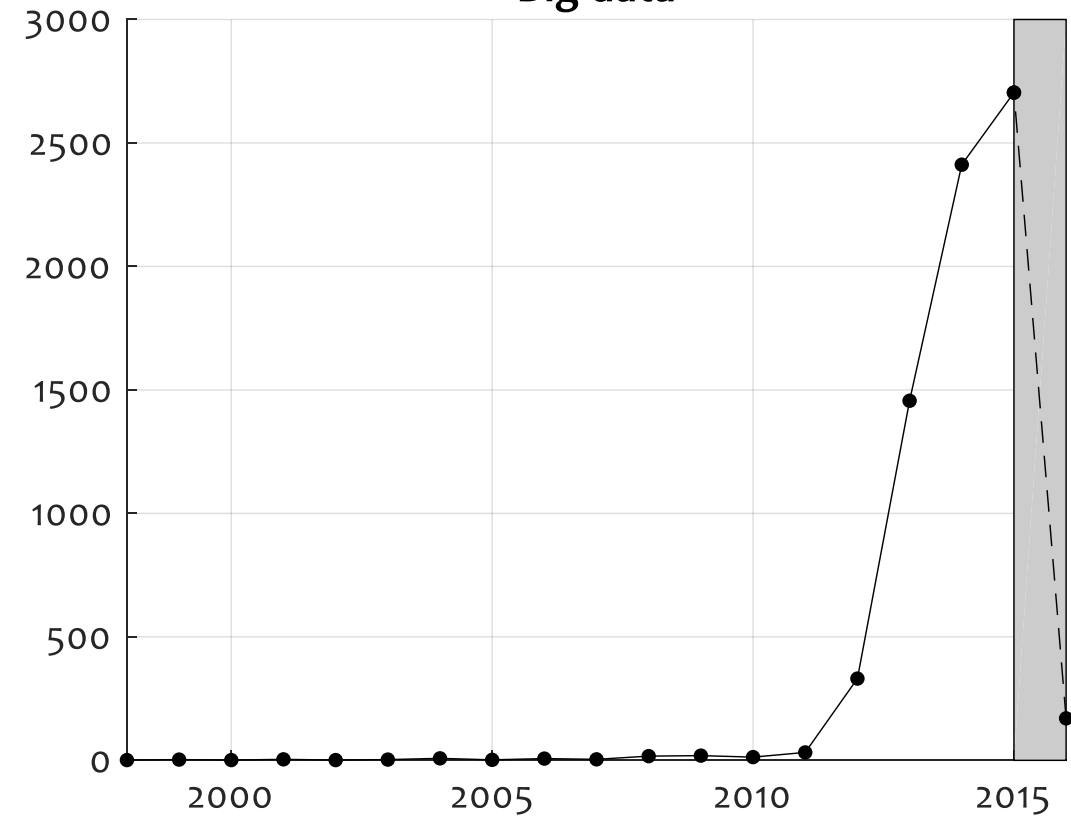
Doing research these days...

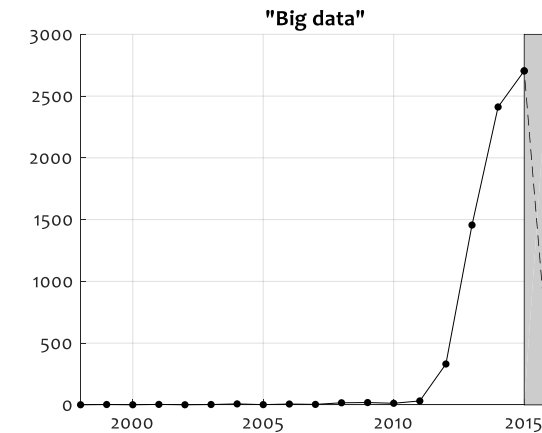
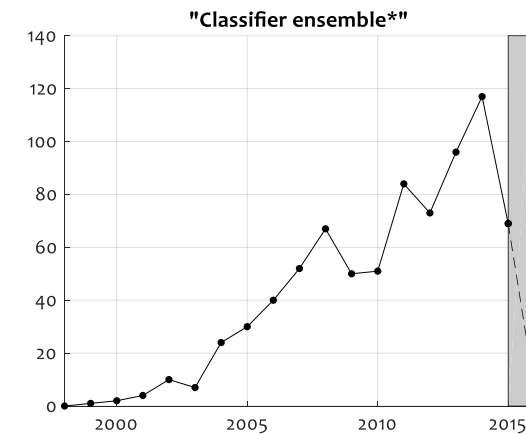
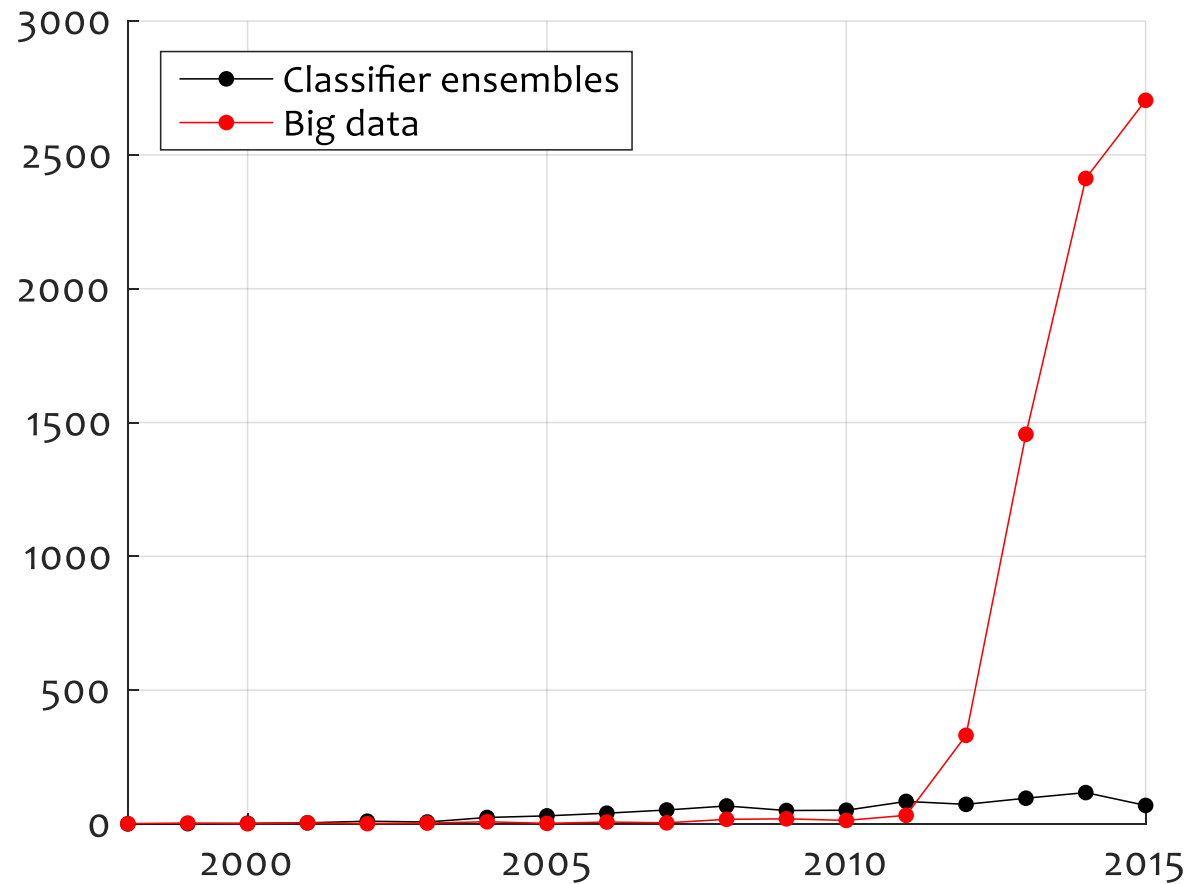


"Classifier ensemble*"



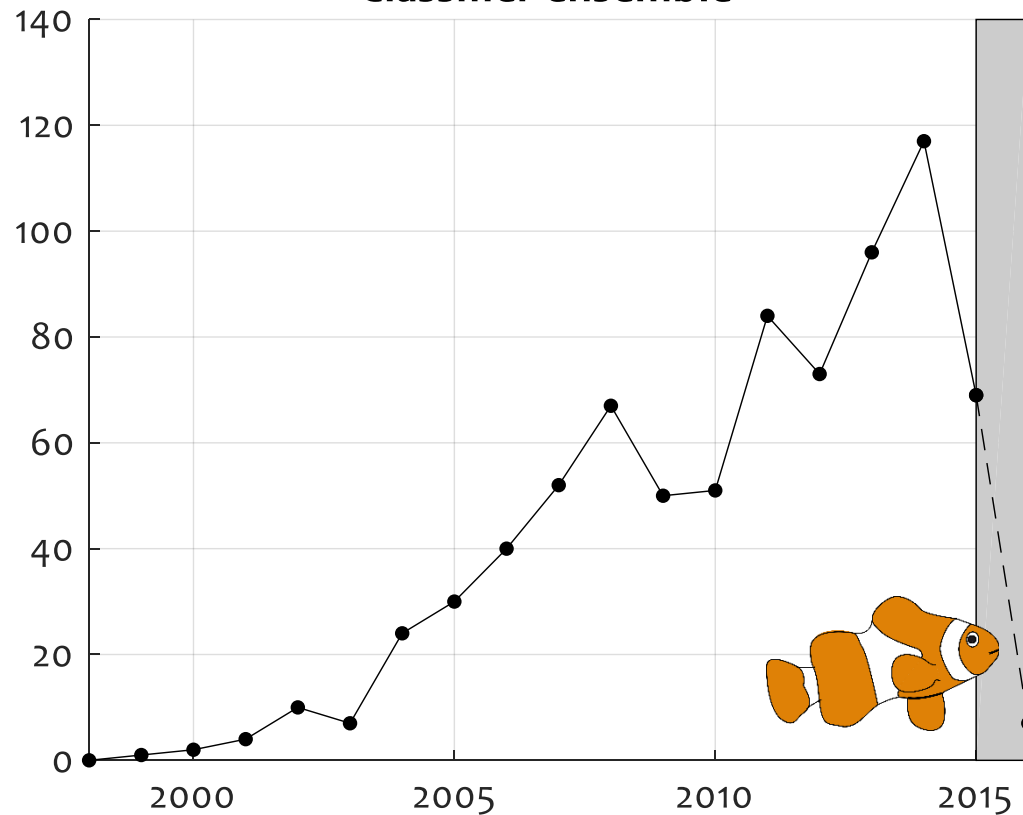
"Big data"



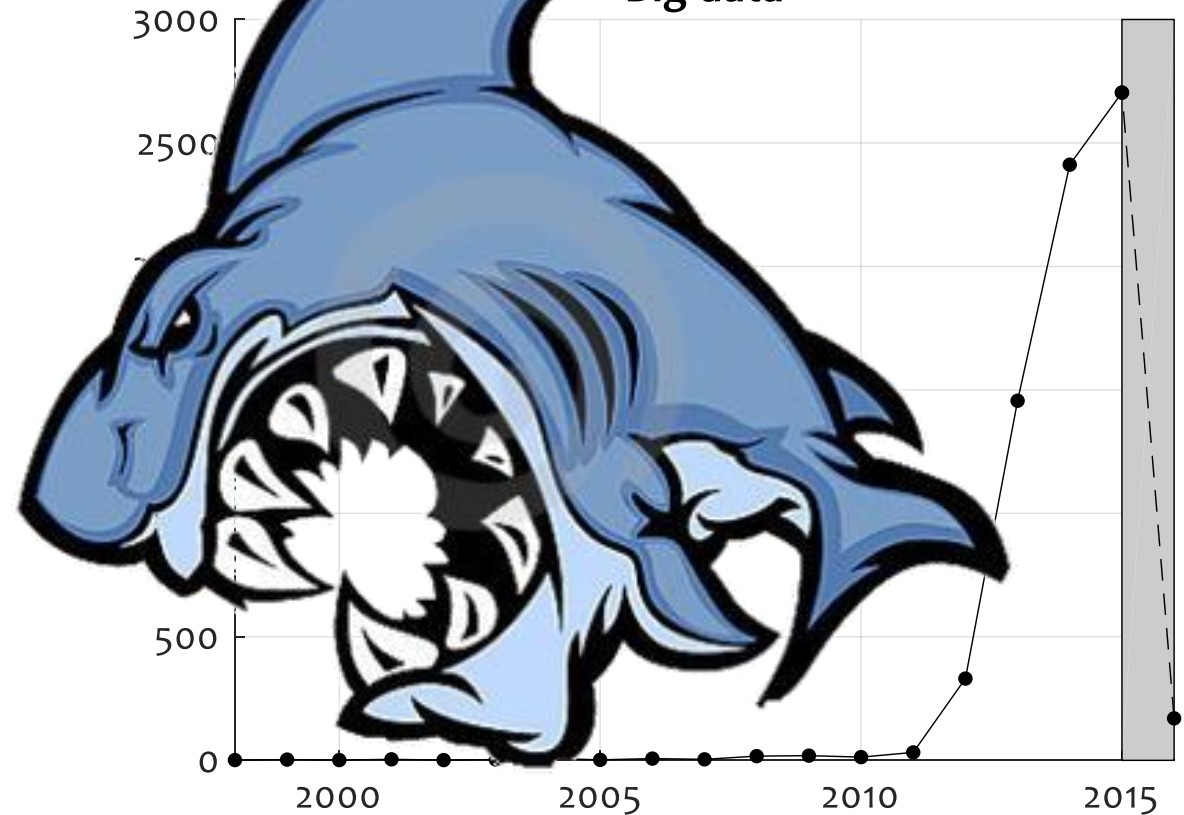


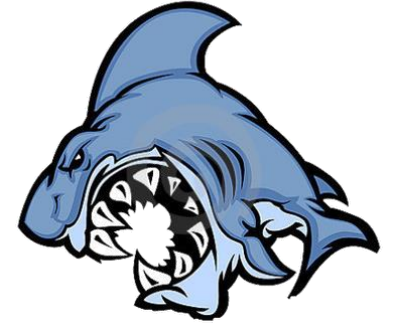


"Classifier ensemble*"



"Big data"





CLOUD

Yahoo releases largest ever machine learning dataset to research

Martin Anderson Thu 14 Jan 2016 4.05pm

110 billion events



1.5 terabytes zipped



Ronald Fisher



The FAMOUS Iris Data



09 February 2016

"iris data"

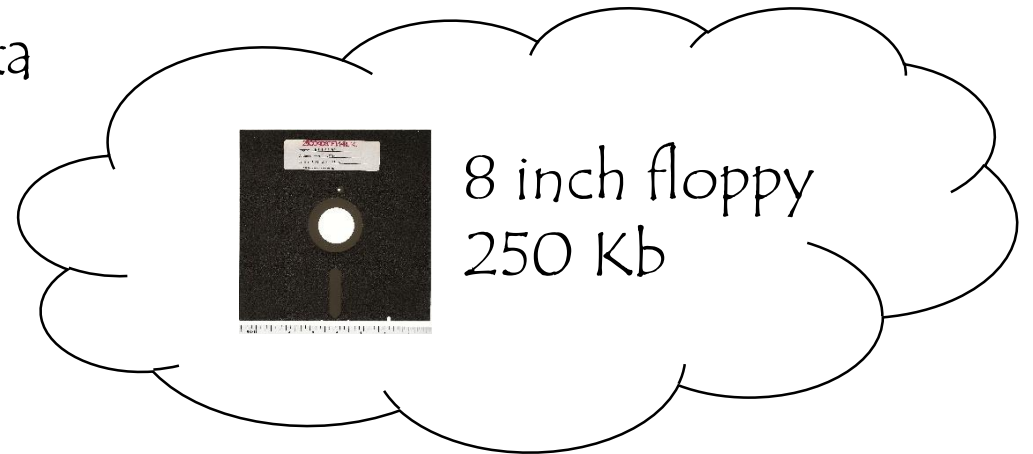
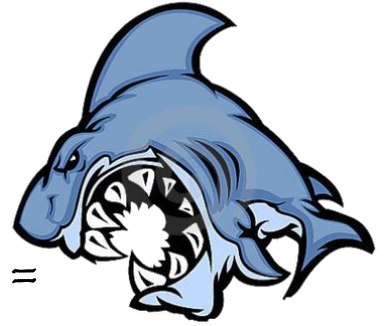
568 hits

150 data points
4 features + labels

$150 \times (4+1) \times 8 \text{ (bytes)} =$

6Kb

k-i-l-o-b-y-t-e-s, that is



Dre-e-e-am
dream dream dreaaam...



1.44 Mb
Wow!!!





Let's say, generously, 1.5 Mb

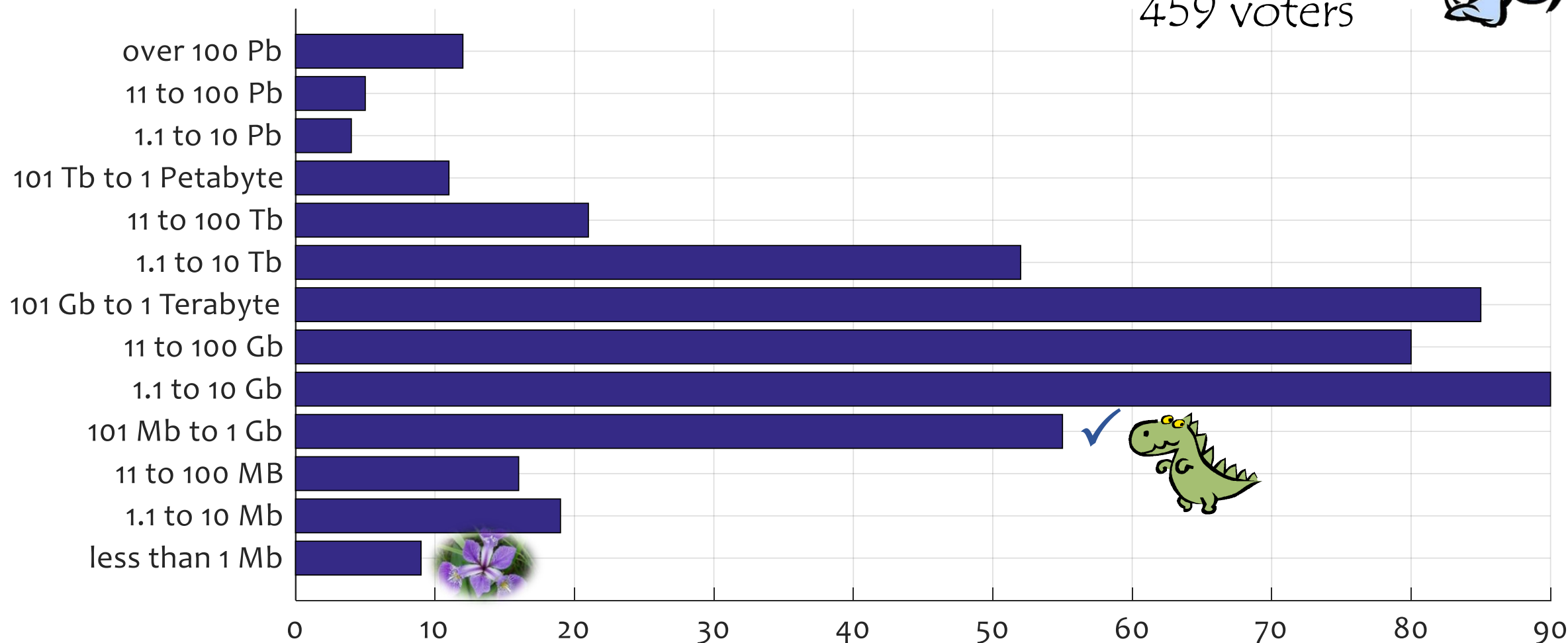
Petabyte of storage is about 666,666,667 floppies

KDnuggets Home » Polls » largest dataset analyzed / data mined? Poll (Aug 2015)

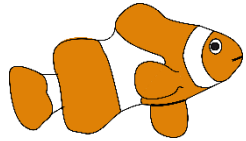


What was the largest dataset you analysed / data mined?

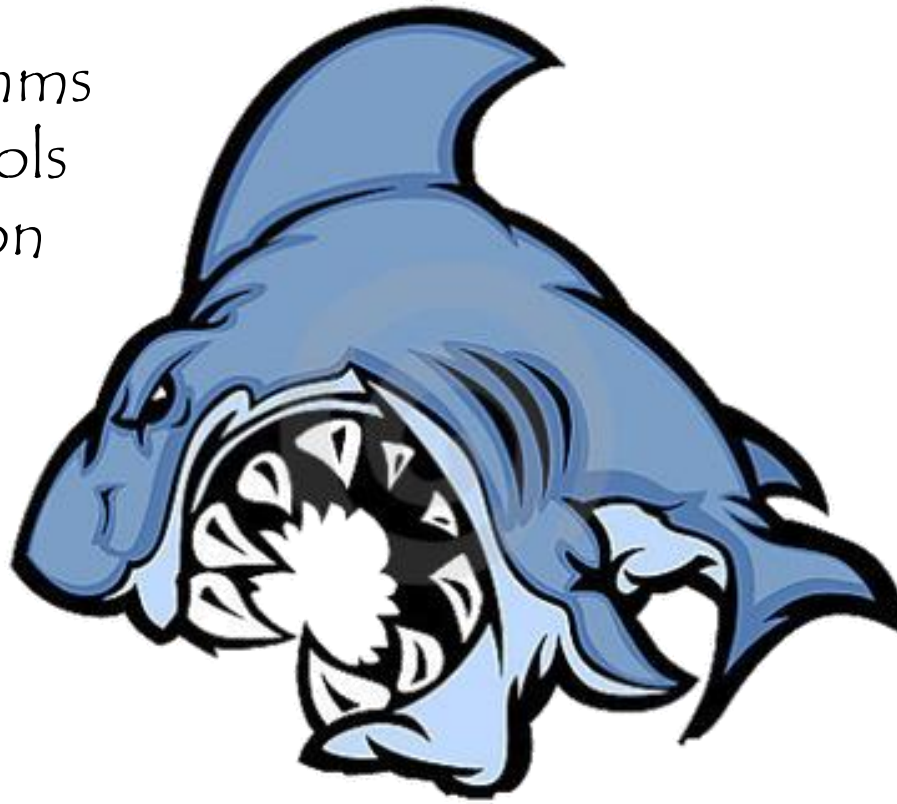
459 voters



Pattern recognition algorithms
Training and testing protocols
Clever density approximation
Ingenious models



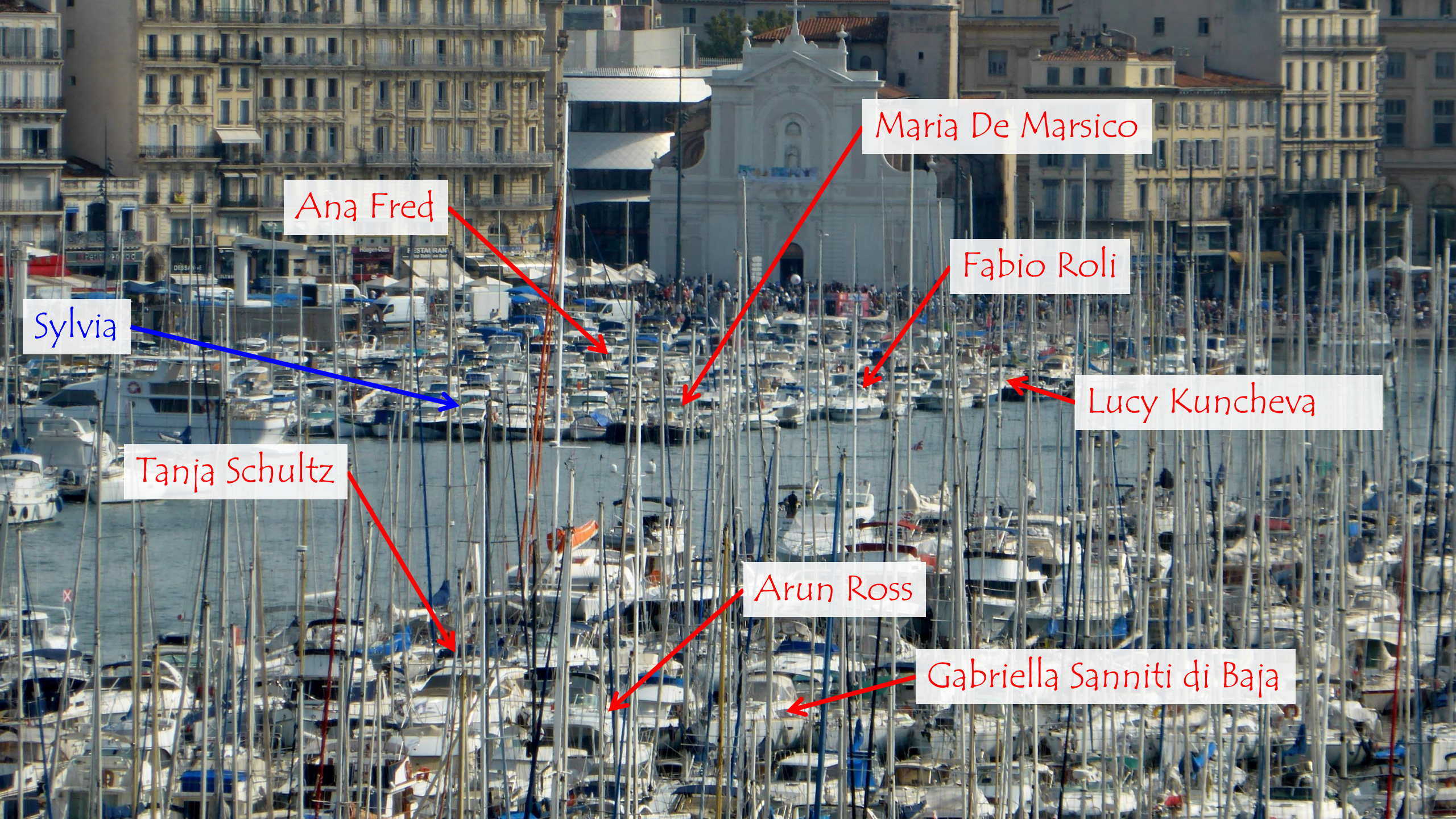
bye-bye...



A bit depressing...

HELLO

Data management and organisation
Efficient storage
Distributed computing
Fast computing
Optimisation / search
Computer clusters



Ana Fred

Sylvia

Tanja Schultz

Maria De Marsico

Fabio Roli

Lucy Kuncheva

Arun Ross

Gabriella Sanniti di Baja

Enjoying the ride but kind of... insignificant





All the Giants



You...



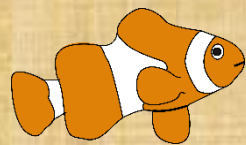
Supercomputer

What chance do **you** have with your little...

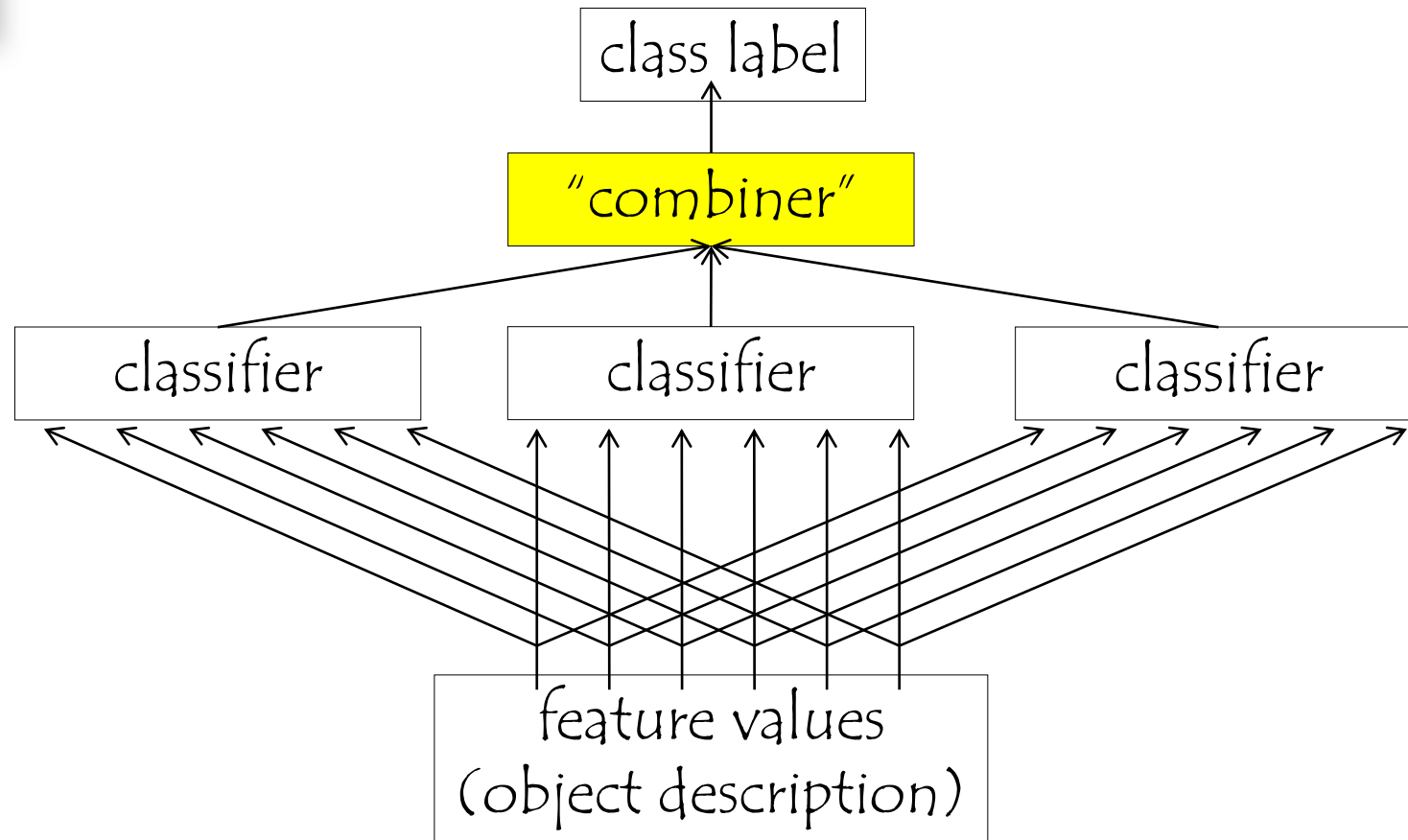


You...

Classifier Ensembles?



Classifier ensembles



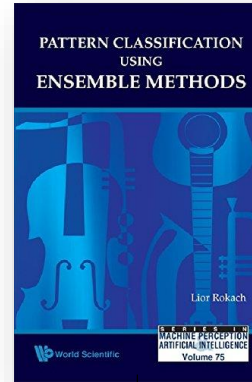
Classifier ensembles

VERY well liked classification approach. And this is why:

1. Typically better than the individual ensemble members and other individual classifiers.
2. And the above is enough 😊

Classifier ensembles

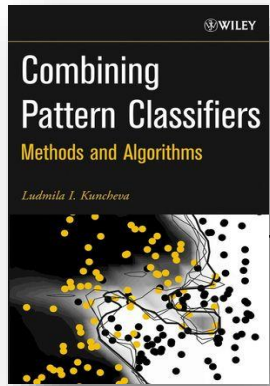
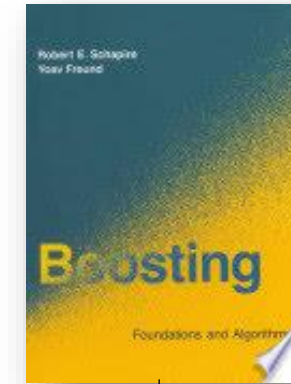
Lior Rokach
Pattern Classification Using
Ensemble Methods
World Scientific
2010



Zhi-Hua Zhou
Ensemble
Methods:
Foundations and
Algorithms
Chapman &
Hall/Crc , 2012



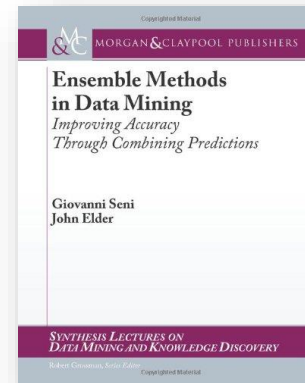
Robert E. Schapire and
Yoav Freund
Boosting:
Foundations
and
Algorithms
MIT Press,
2014



2004

Ludmila Kuncheva
Combining Pattern Classifiers.
Methods and Algorithms
Wiley, 2004

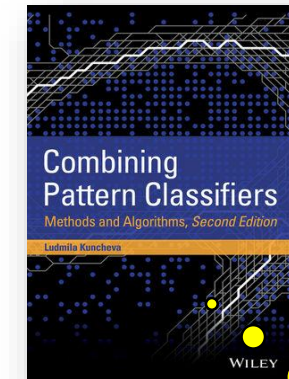
2010



Giovanni Seni and John F. Elder
Ensemble Methods in Data Mining
Morgan & Claypool Publishers, 2010

2012

2014



Second
edition

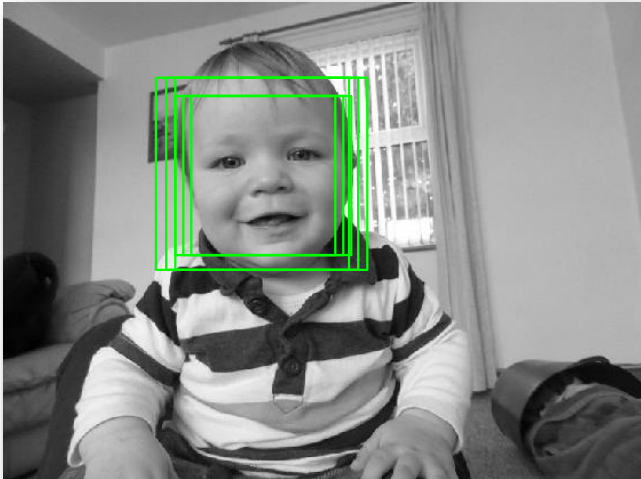
Classifier ensembles



2009 – Winner 1,000,000 USD – an **ensemble** method

US Patents Satellite classifier **ensemble**
US 7769701 B2

Methods for feature selection using classifier
ensemble based genetic algorithms
US 8762303 B



cited 12,403
times by 19/02/2016
(Google Scholar)

Rapid Object Detection using a Boosted Cascade of Simple Features

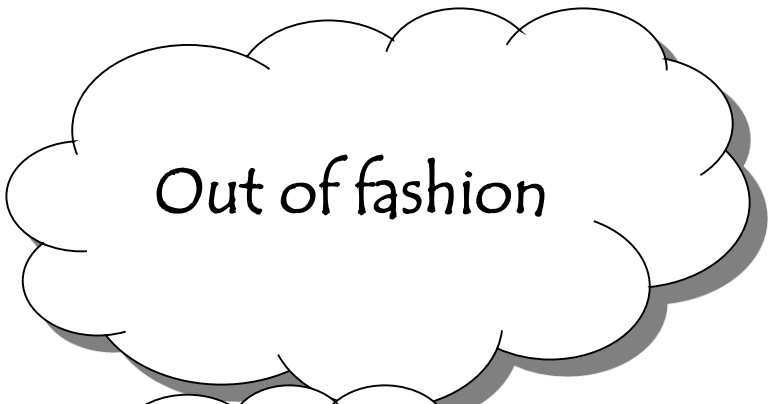
Paul Viola
viola@merl.com
Mitsubishi Electric Research Labs
201 Broadway, 8th FL
Cambridge, MA 02139

Michael Jones
mjones@crl.dec.com
Compaq CRL
One Cambridge Center
Cambridge, MA 02142

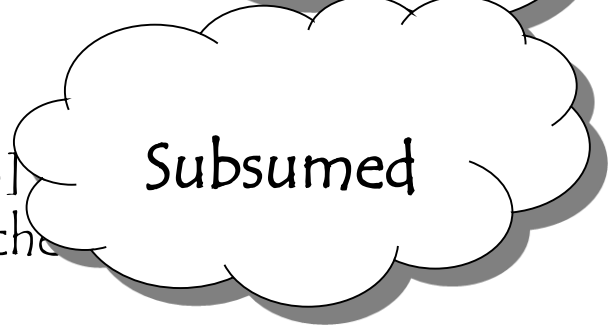
AdaBoost

combination of multiple classifiers [Lam95,Woods97,Xu92,Kittler98]
classifier fusion [Cho95,Gader96,Grabisch92,Keller94,Bloch96]
mixture of experts [Jacobs91,Jacobs95,Jordan95,Nowlan91]
committees of neural networks [Bishop95,Drucker94]
consensus aggregation [Benediktsson92,Ng92,Benediktsson97]
voting pool of classifiers [Battiti94]
dynamic classifier selection [Woods97]
composite classifier systems [Dasarathy78] ← oldest
classifier ensembles [Drucker94,Filippi94,Sharkey99]
bagging, boosting, arcing, wagging [Sharkey99]
modular systems [Sharkey99] ← oldest
collective recognition [Rastrigin81,Barabash83]
stacked generalization [Wolpert92]
divide-and-conquer classifiers [Chiang94] ← fanciest
pandemonium system of reflective agents [Smieja96]
change-glasses approach to classifier selection [KunchevaPRL93]
etc.

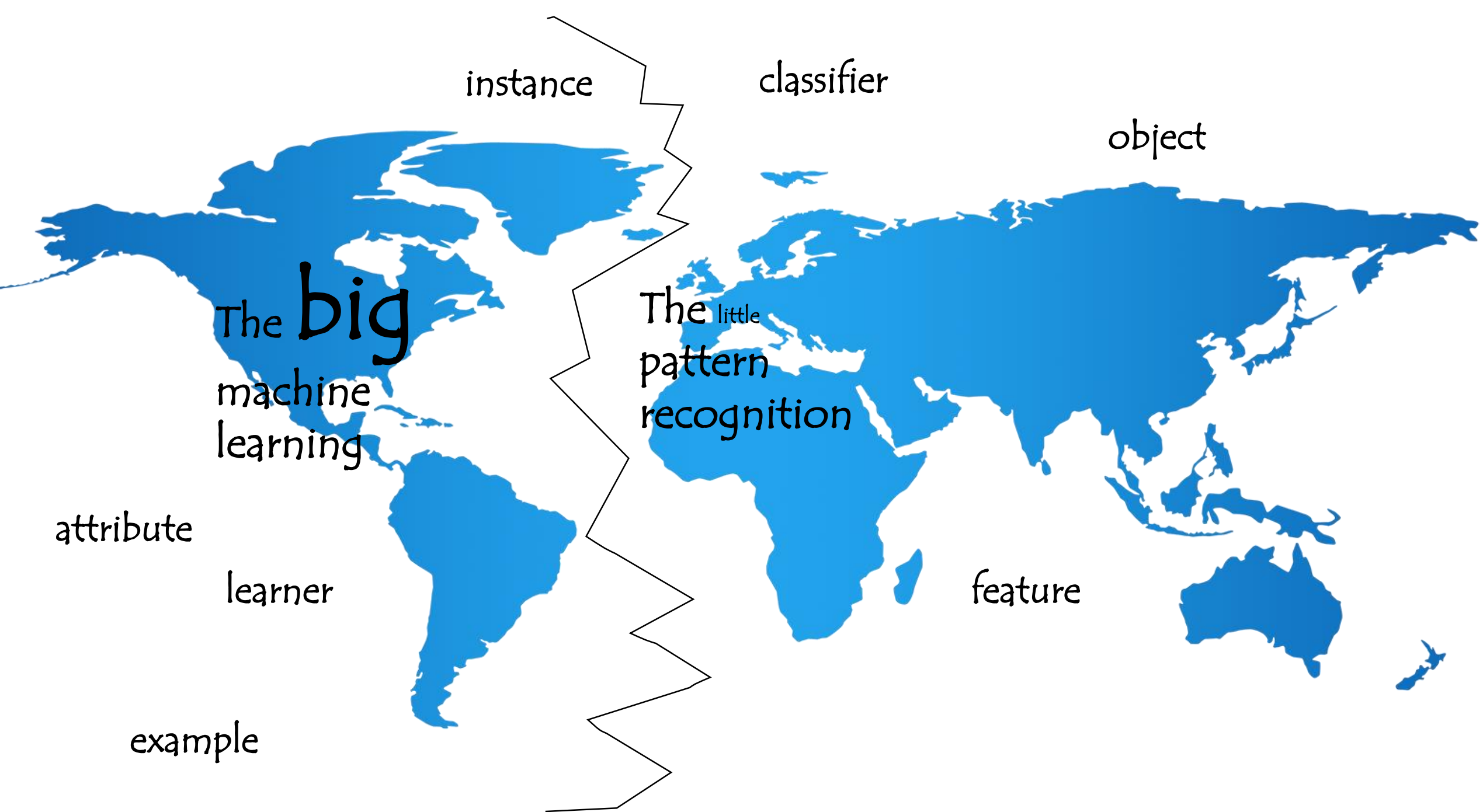
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classifier fusion [Cho95,Gader96,Grabisch92,Keller94,Bloch96]
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divide-and-conquer classifiers [Chiang94]
pandemonium system of reflective agents [Smieja96]
change-glasses approach to classifier selection [Kunchev96]
etc.



Out of fashion



Subsumed



instance

classifier

object

The **big**
machine
learning

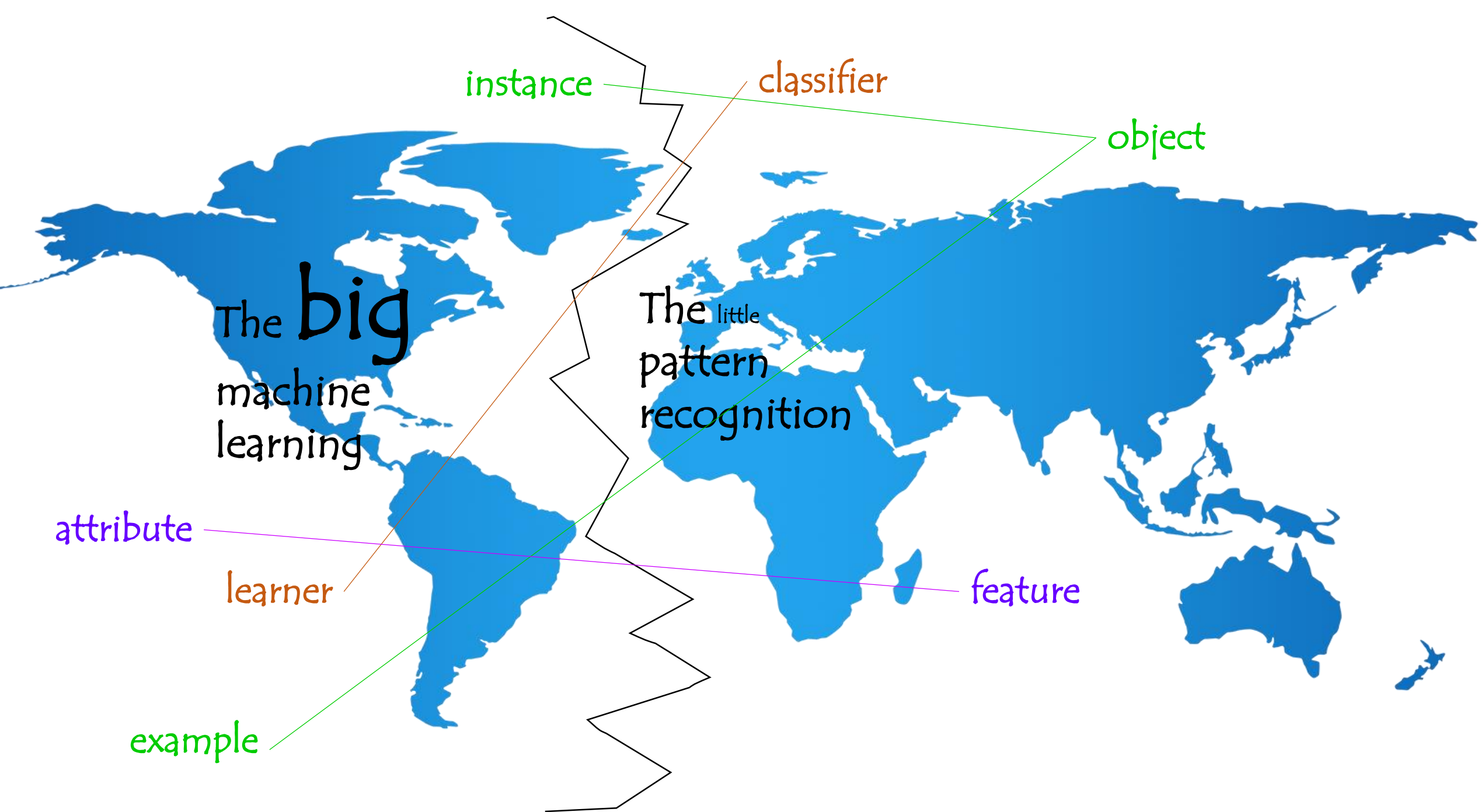
The little
pattern
recognition

attribute

learner

example

feature





International Workshops on
Multiple Classifier Systems
2000 – 2014 – continuing



How do we design/build/train
classifier ensembles?

Building ensembles

Levels of questions

A Combination level

- selection or fusion?
- voting or another combination method?
- trainable or non-trainable combiner?
- and why not another classifier?

B Classifier level

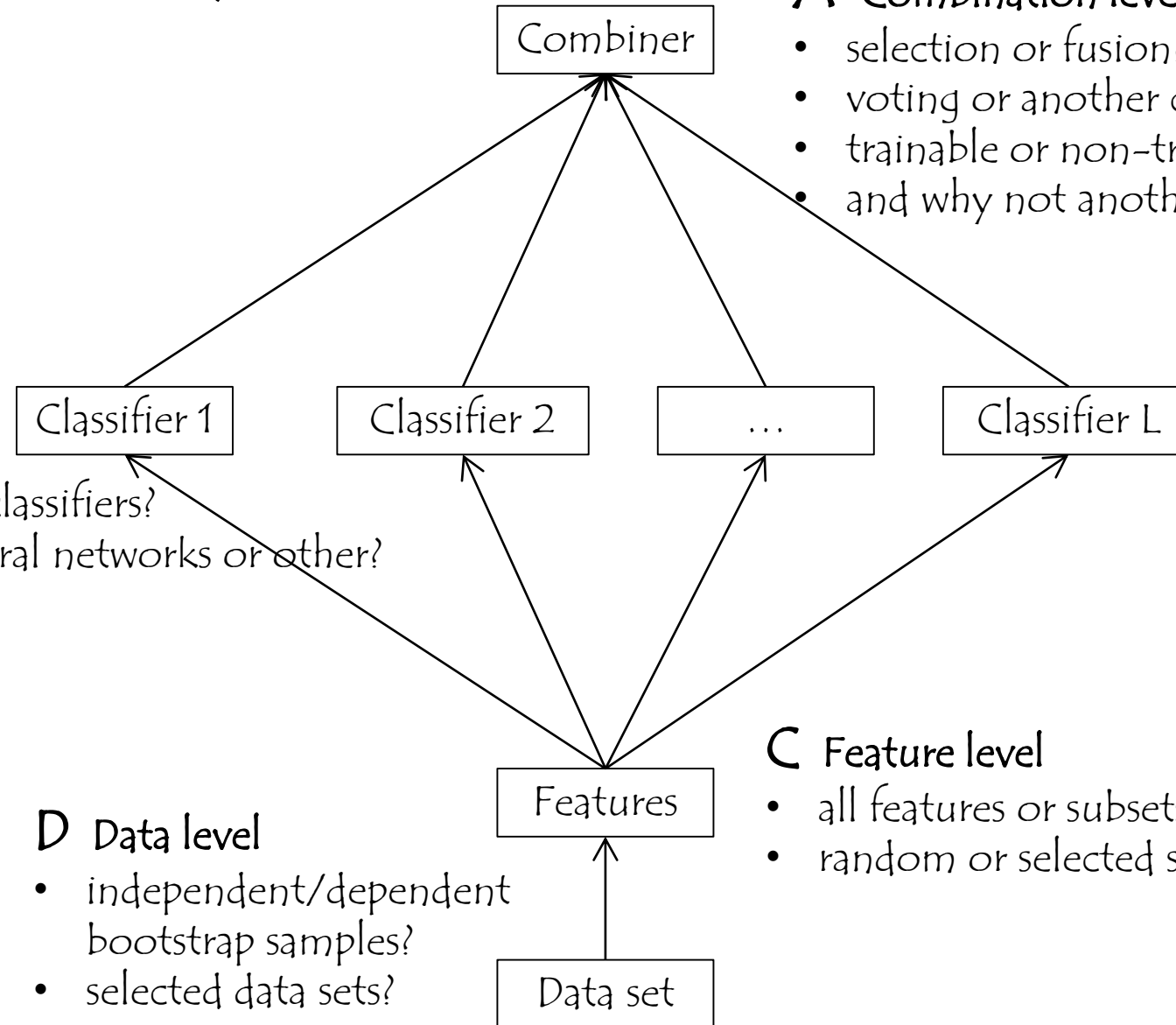
- same or different classifiers?
- decision trees, neural networks or other?
- how many?

D Data level

- independent/dependent bootstrap samples?
- selected data sets?

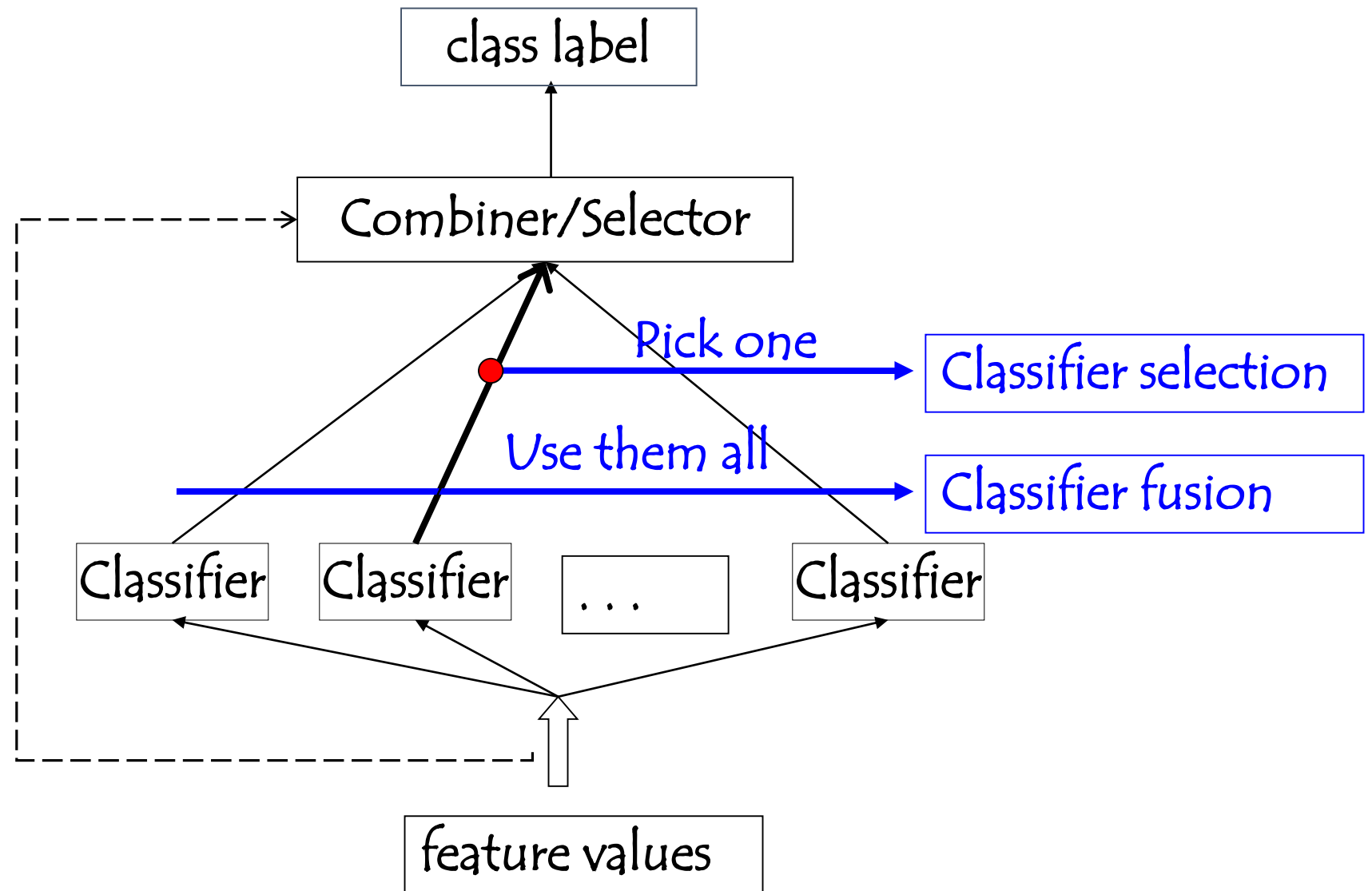
C Feature level

- all features or subsets of features?
- random or selected subsets?



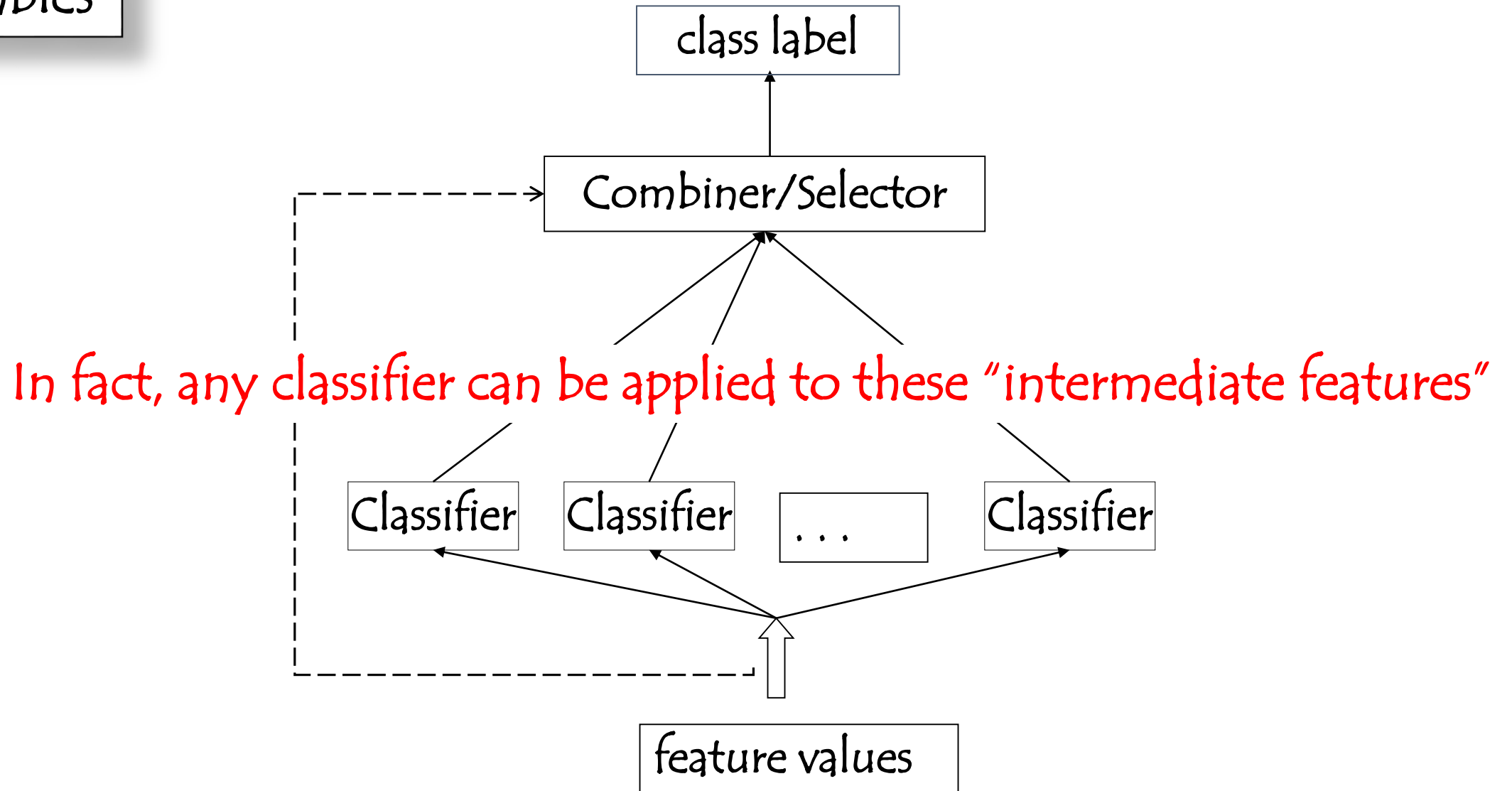
Building ensembles

The two strategies: fusion versus selection



Building ensembles

The two strategies: fusion versus selection



Classifier ensembles: the "CLASSICS"

- Bagging

sample sample sample sample

Independent
bootstrap samples

- Boosting

sample sample sample sample

Dependent
bootstrap samples

- Random Subspace

sample sample sample sample

Independent
feature subsamples

- Random Forest

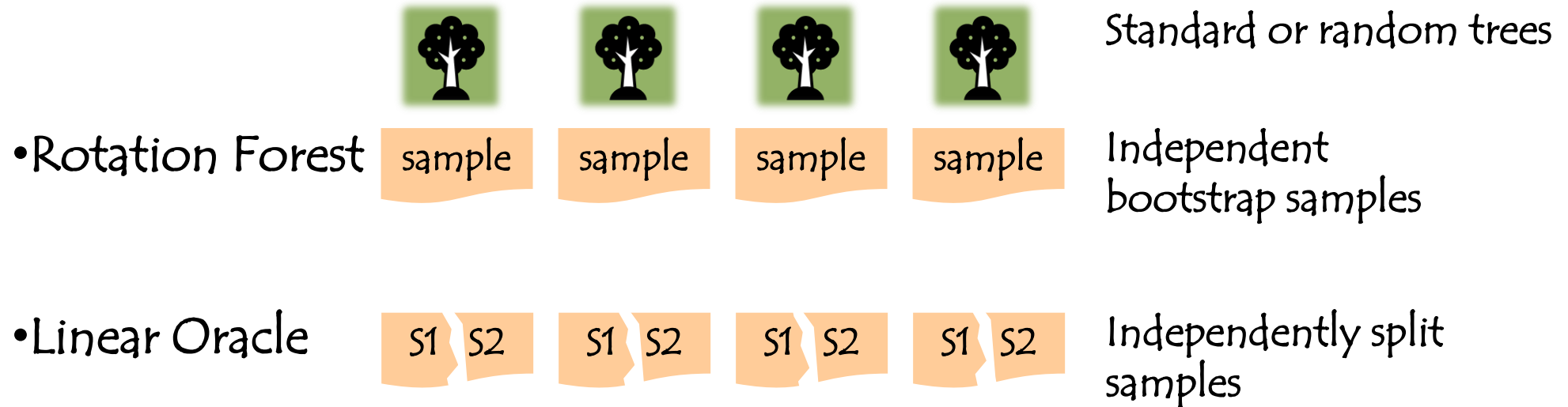


Random trees

sample sample sample sample

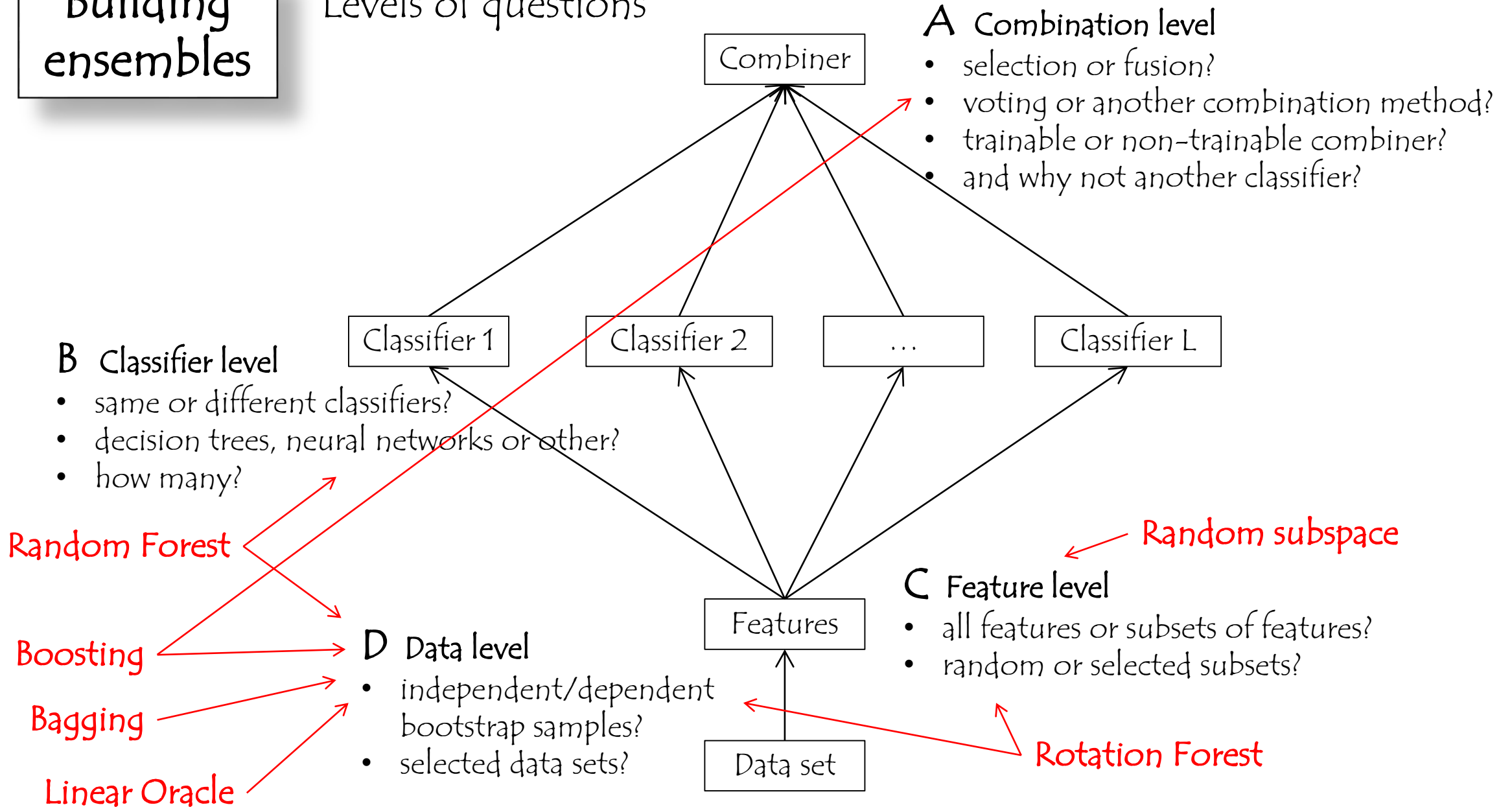
Independent
bootstrap samples

Classifier ensembles: the not-so-classics



Building ensembles

Levels of questions

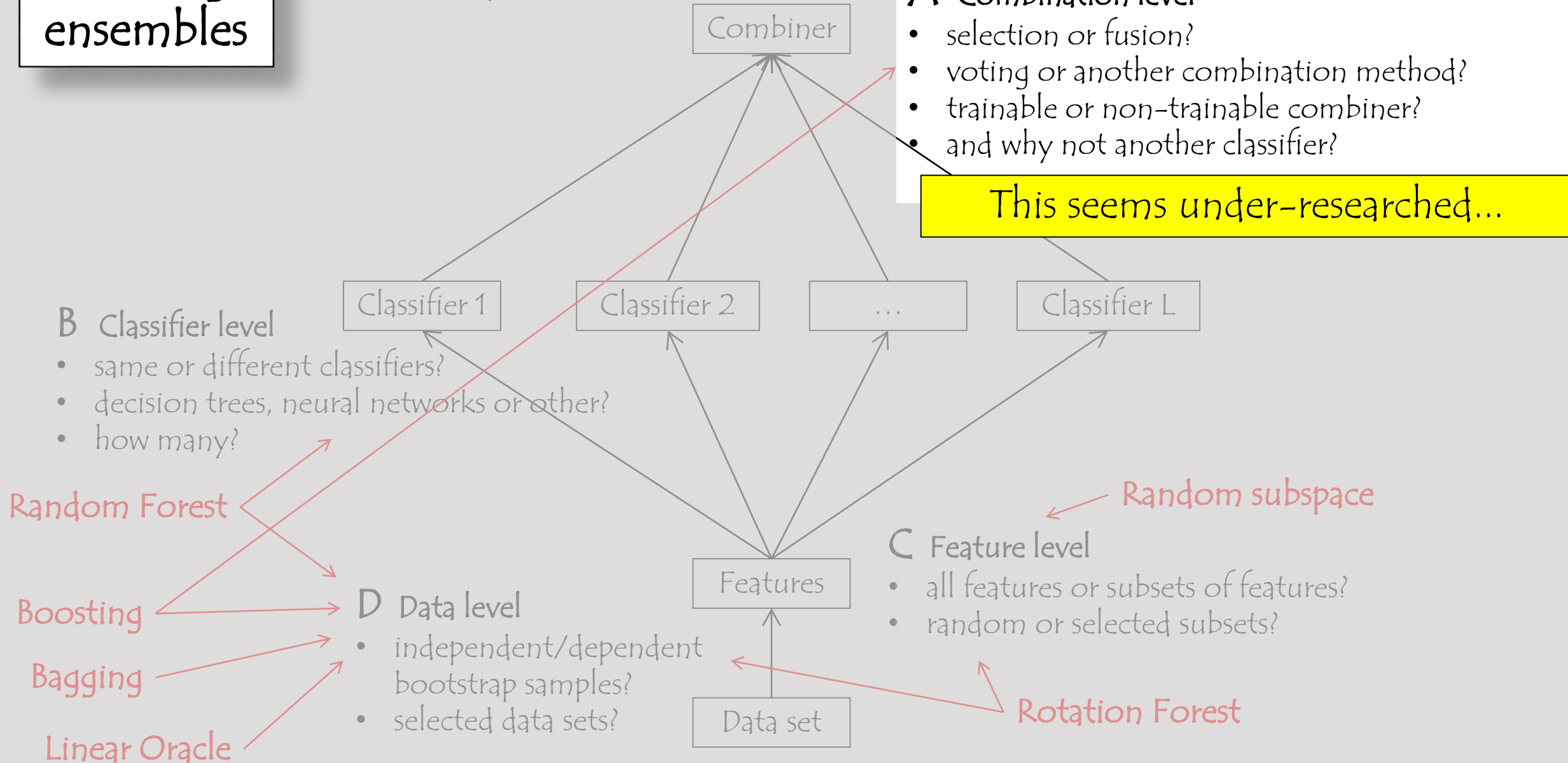


"Anchor" points

1. Combiner

Building ensembles

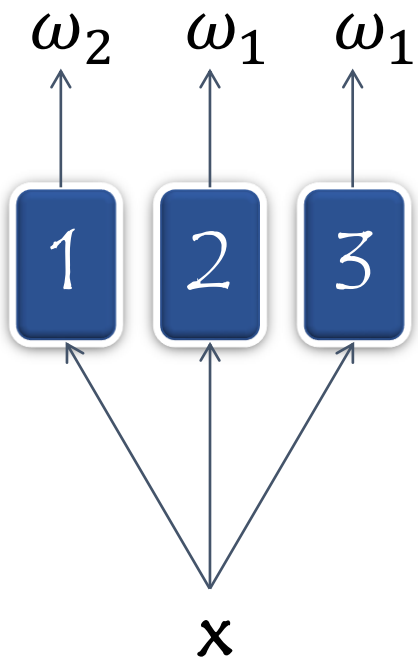
Levels of questions



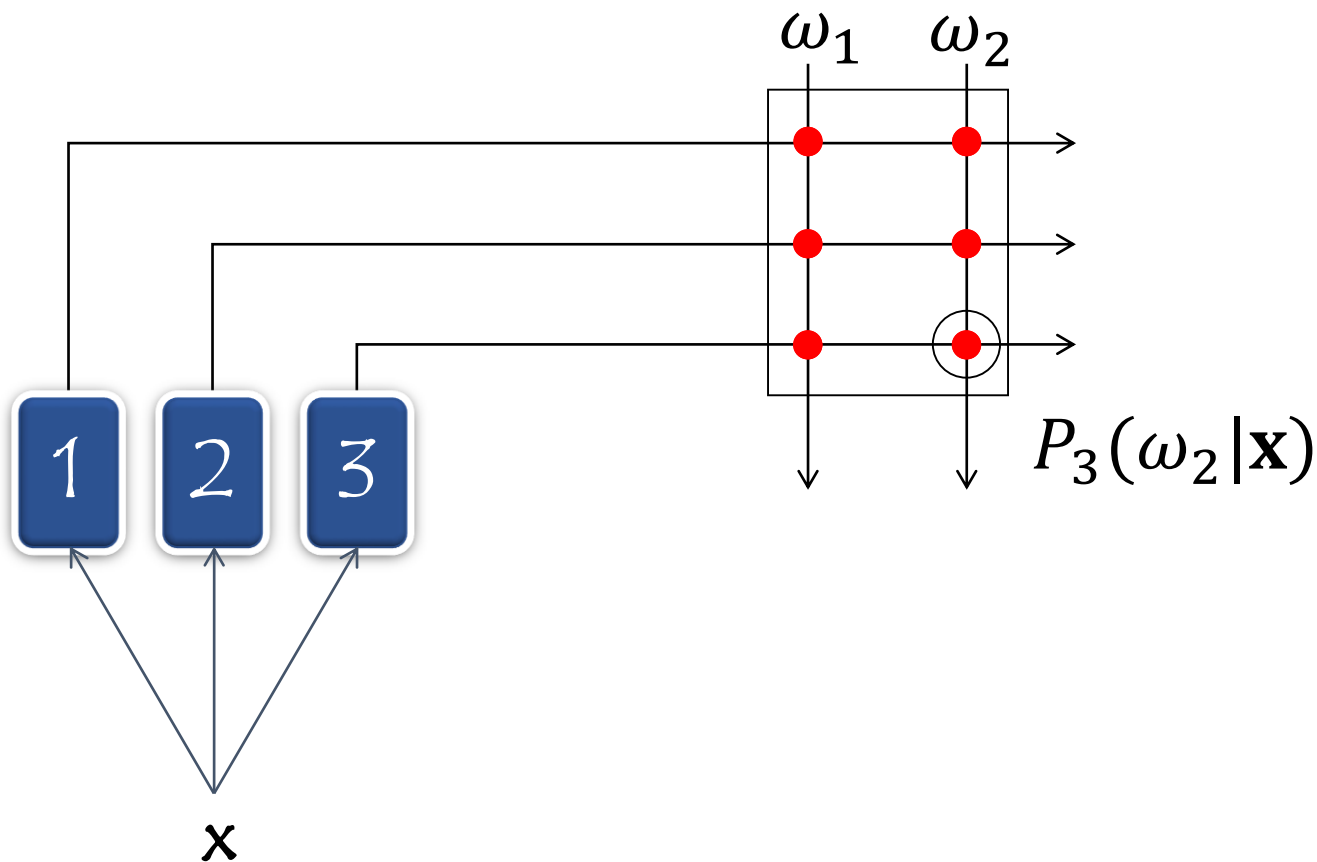
Combiner

Continuous-valued outputs

Label outputs

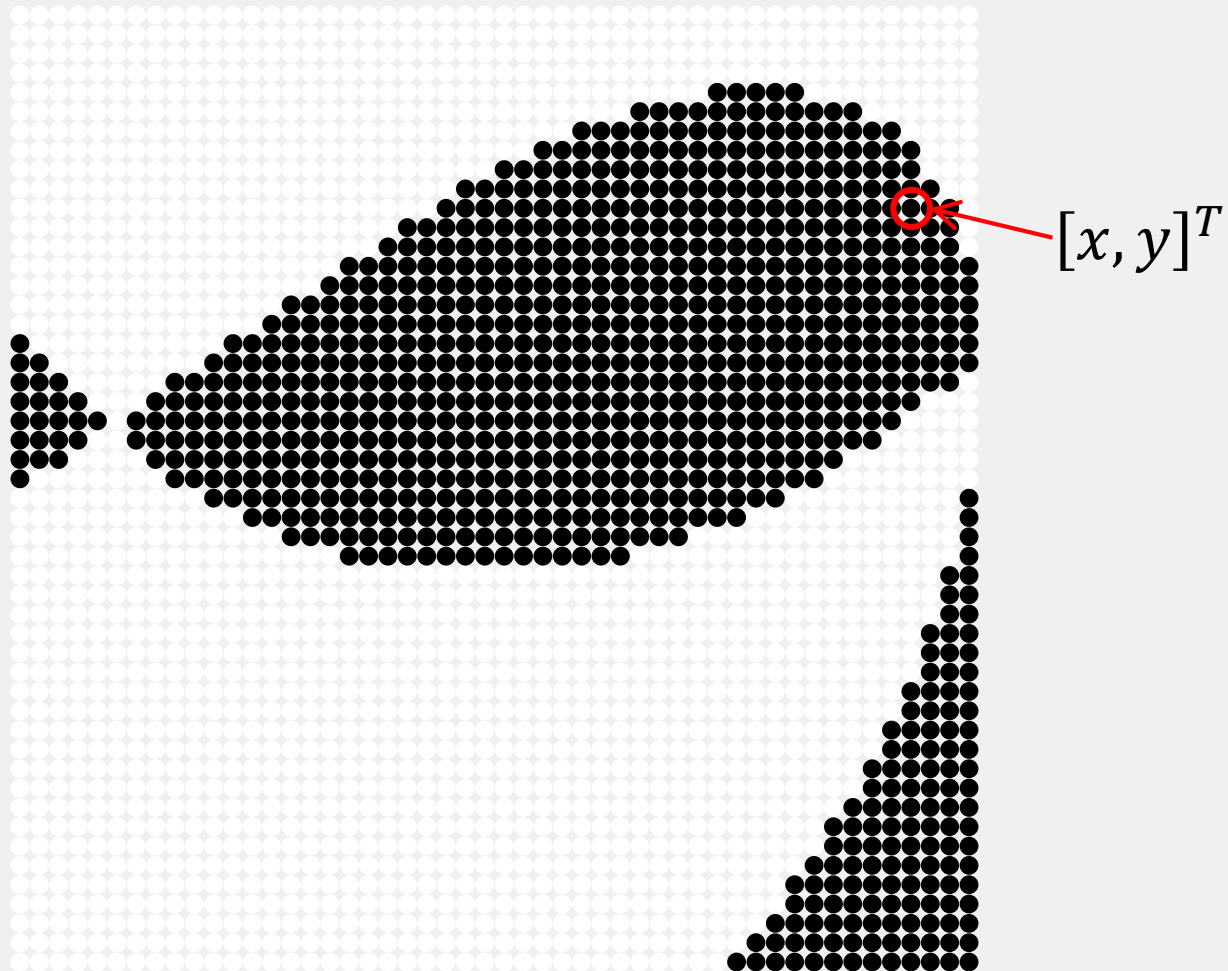


Decision profile



Data set:

Let's call this data "The Tropical Fish" or just the fish data.



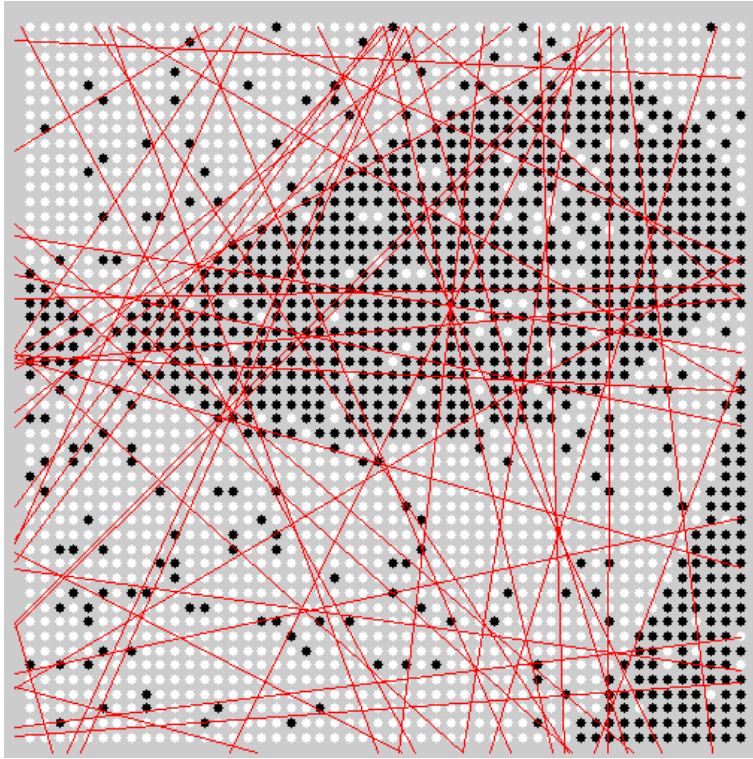
50-by-50 =
2500 objects in 2-d



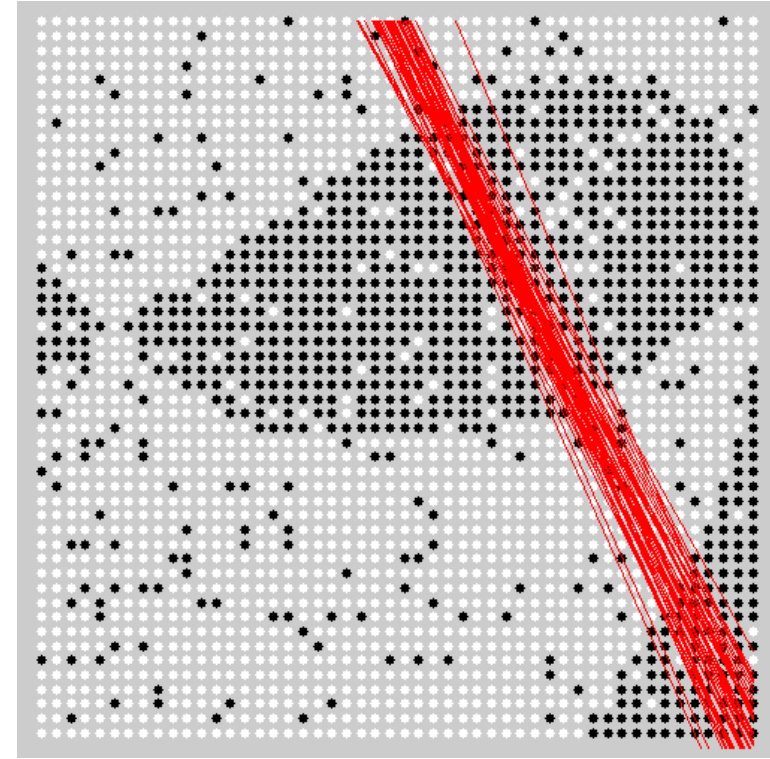
Bayes error rate = 0%

Example: 2 ensembles

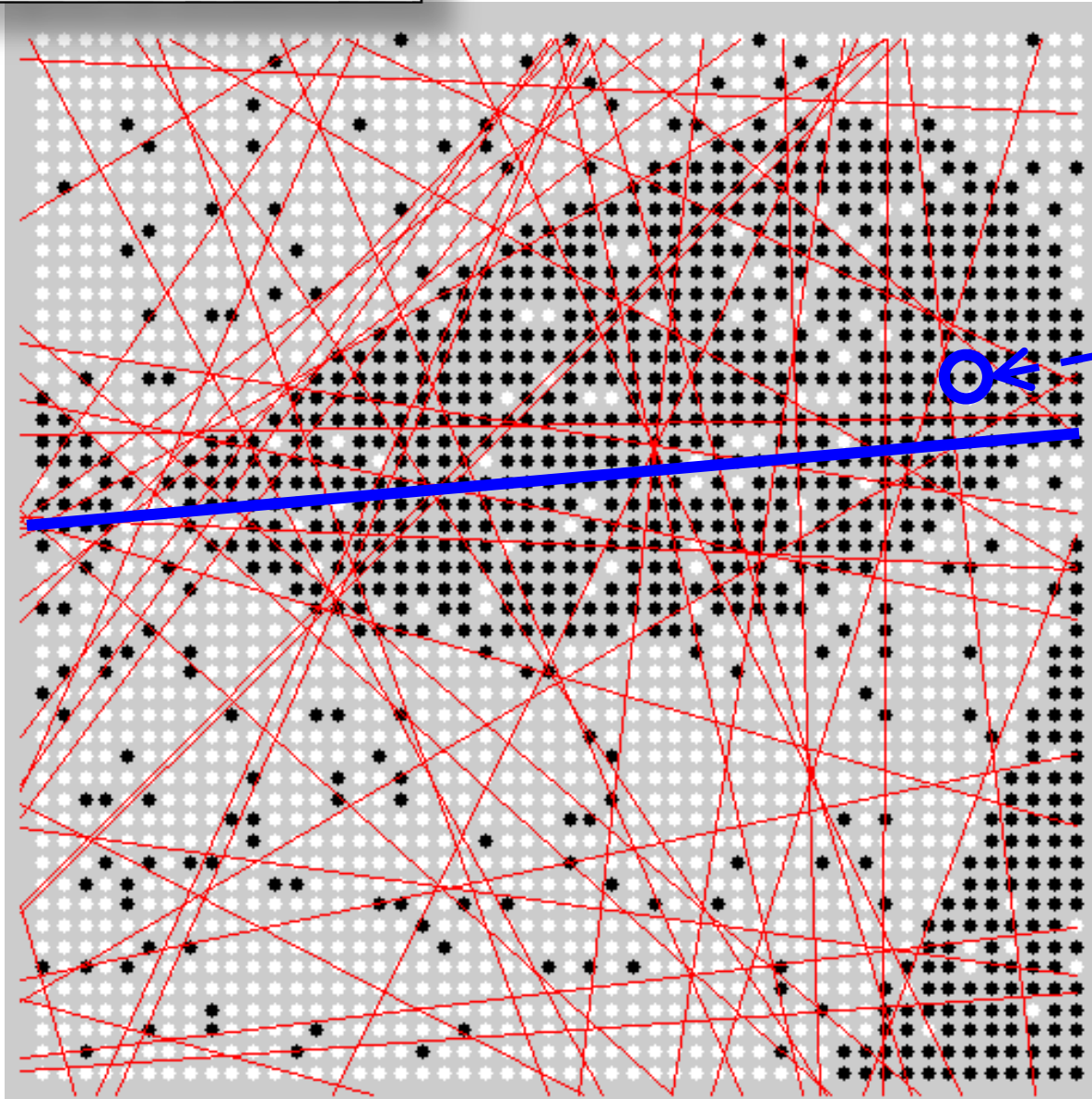
Throw 50 "straws" and label the sides so that the accuracy is greater than 0.5



Train 50 linear classifiers on bootstrap samples



Example: 2 ensembles



Each classifier returns an estimate for class "Fish"

$$P(F | [x, y]^T)$$

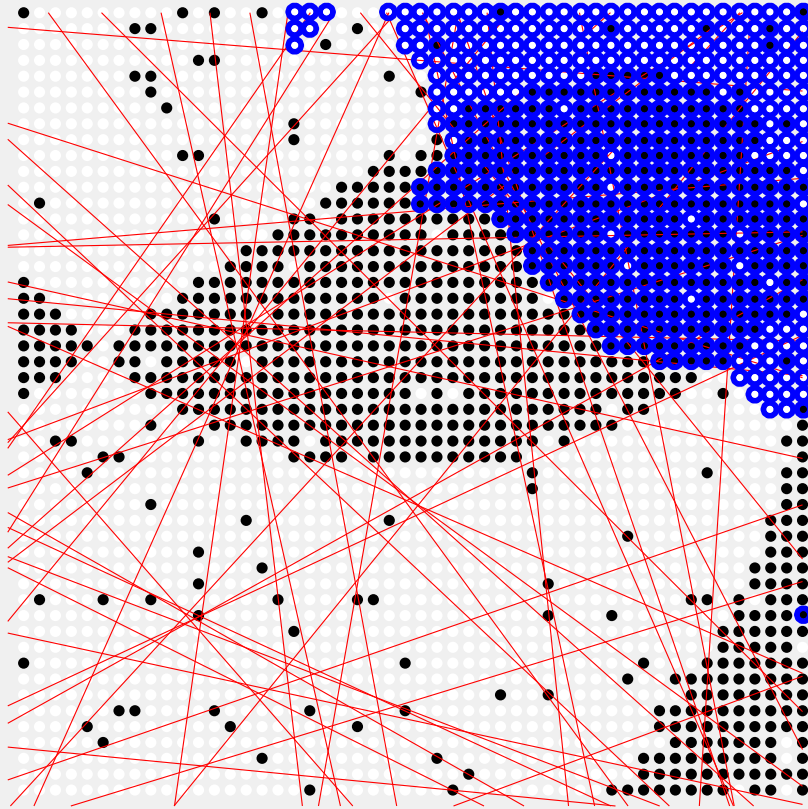
And, of course, we have
 $P(\bar{F} | [x, y]^T) = 1 - P(F | [x, y]^T)$
but we will not need this.

Example: 2 ensembles

5% label noise –
Majority vote

Noise

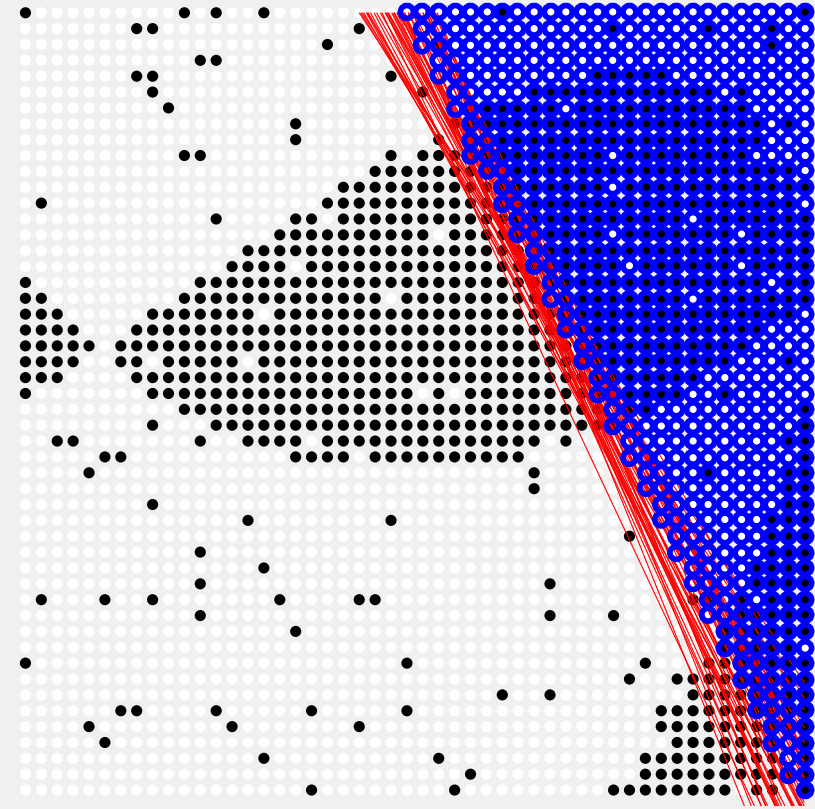
4.98



69.72 %

Combiners

- ☒ Majority vote
- ☐ Average
- ☐ Product
- ☐ Trained Linear
- ☐ Weighted Av...
- ☐ Ridge Regres...
- ☐ Tree Combiner
- ☐ Decision Tem...
- ☐ Naive Bayes
- ☐ BKS



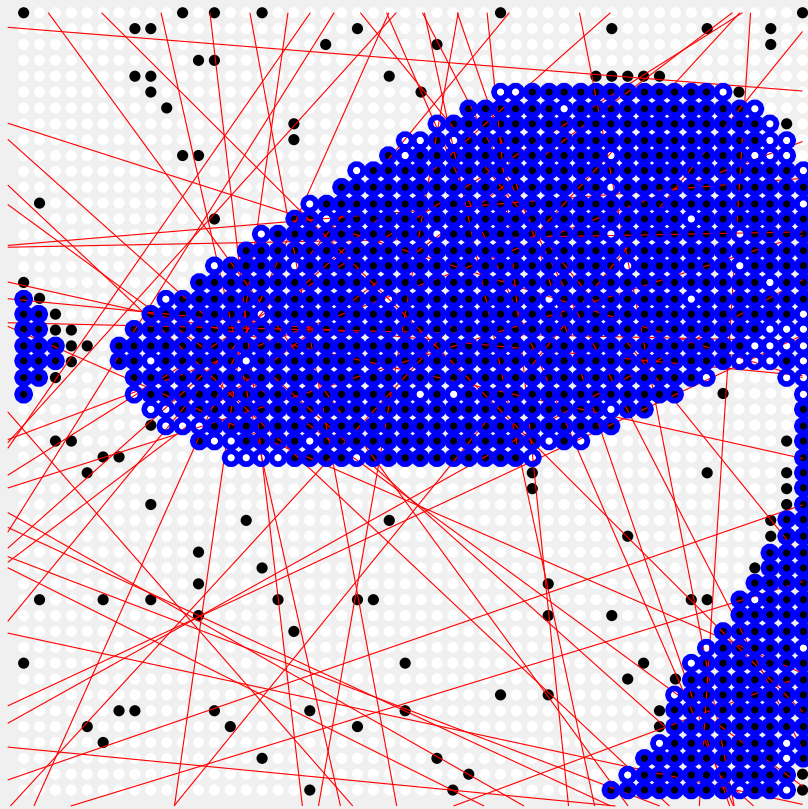
69.60 %

Example: 2 ensembles

5% label noise –
trained linear
combiner

Noise

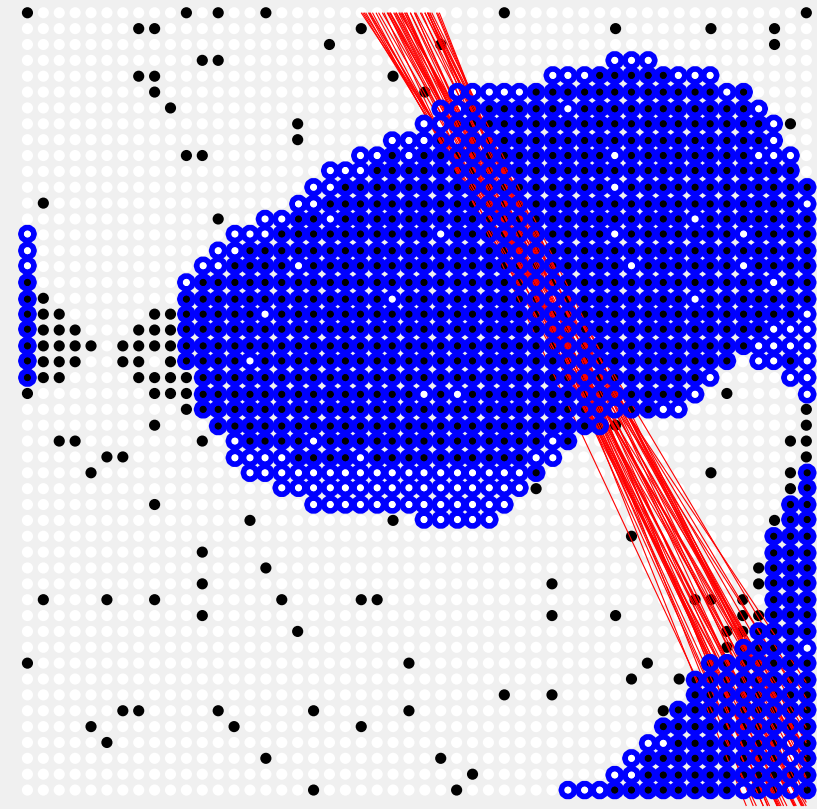
4.98



96.92 %

Combiners

- ☐ Majority vote
- ☐ Average
- ☐ Product
- ☒ Trained Linear
- ☐ Weighted Av...
- ☐ Ridge Regres...
- ☐ Tree Combiner
- ☐ Decision Tem...
- ☐ Naïve Bayes
- ☐ BKS



92.40 %

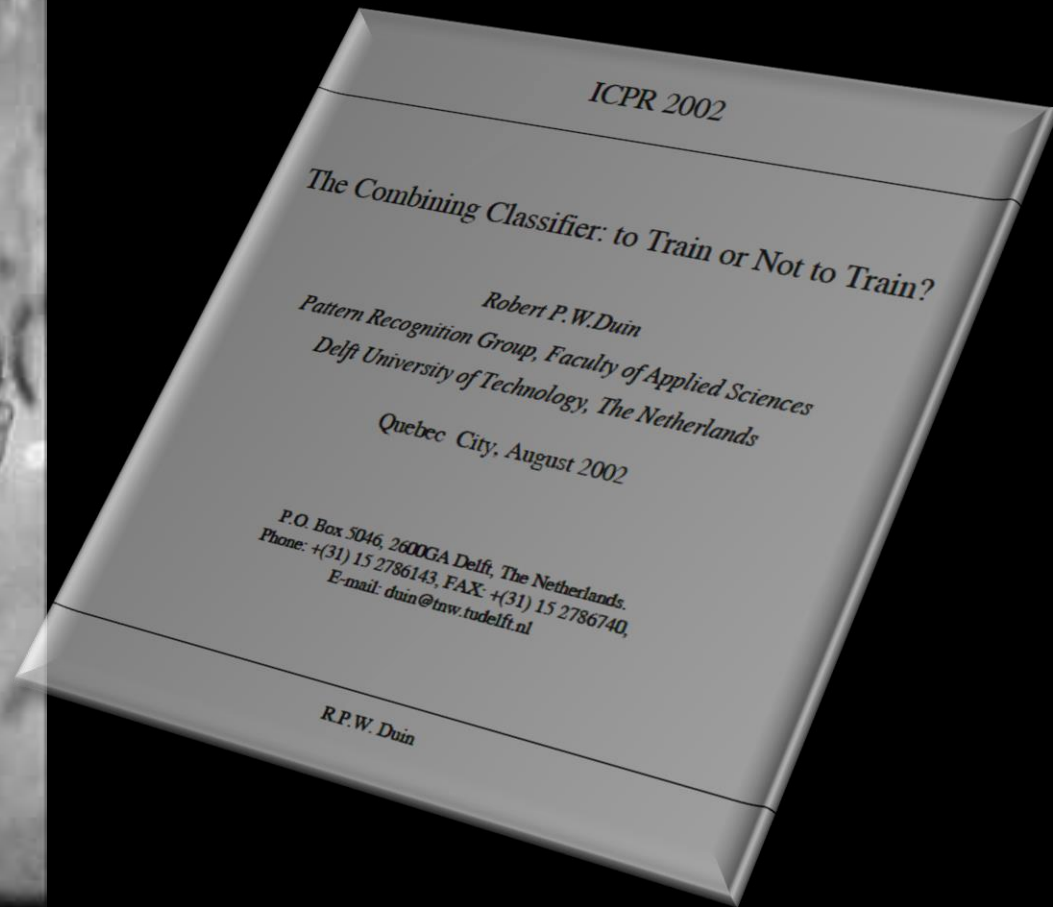
Example: 2 ensembles

What does the example show?

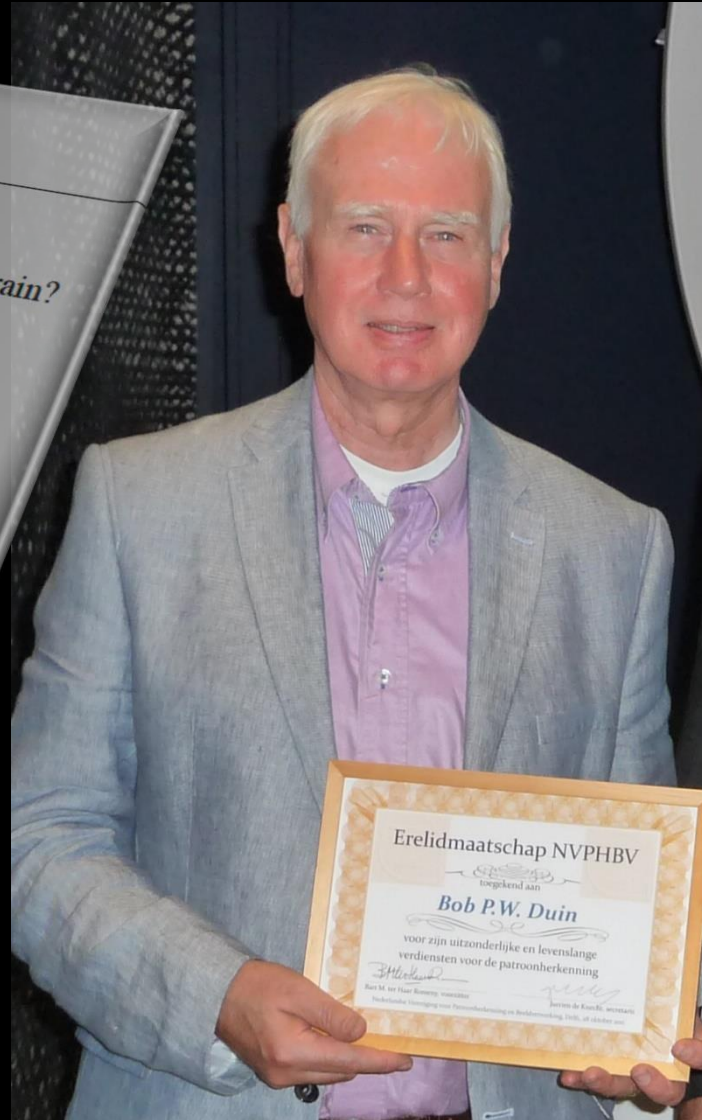
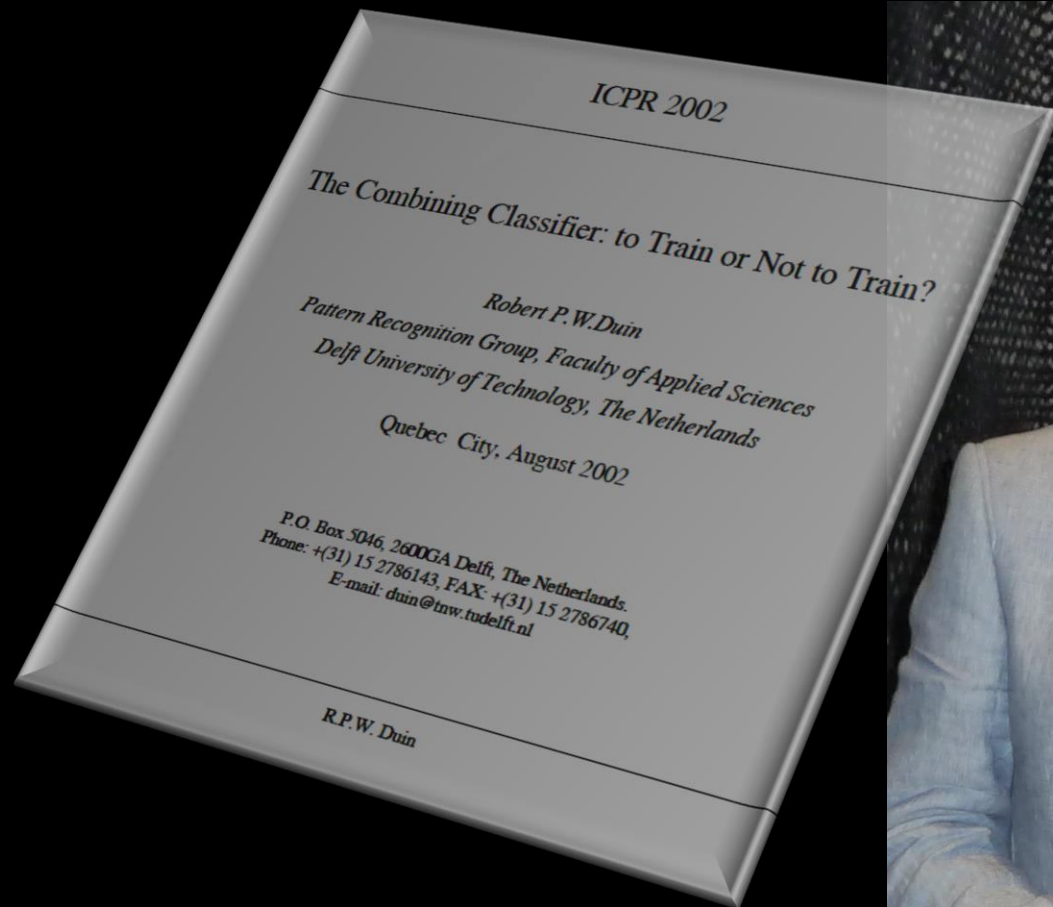
- The combiner matters (a lot)
- The trained combiner works better

However, nothing is as simple as it looks...

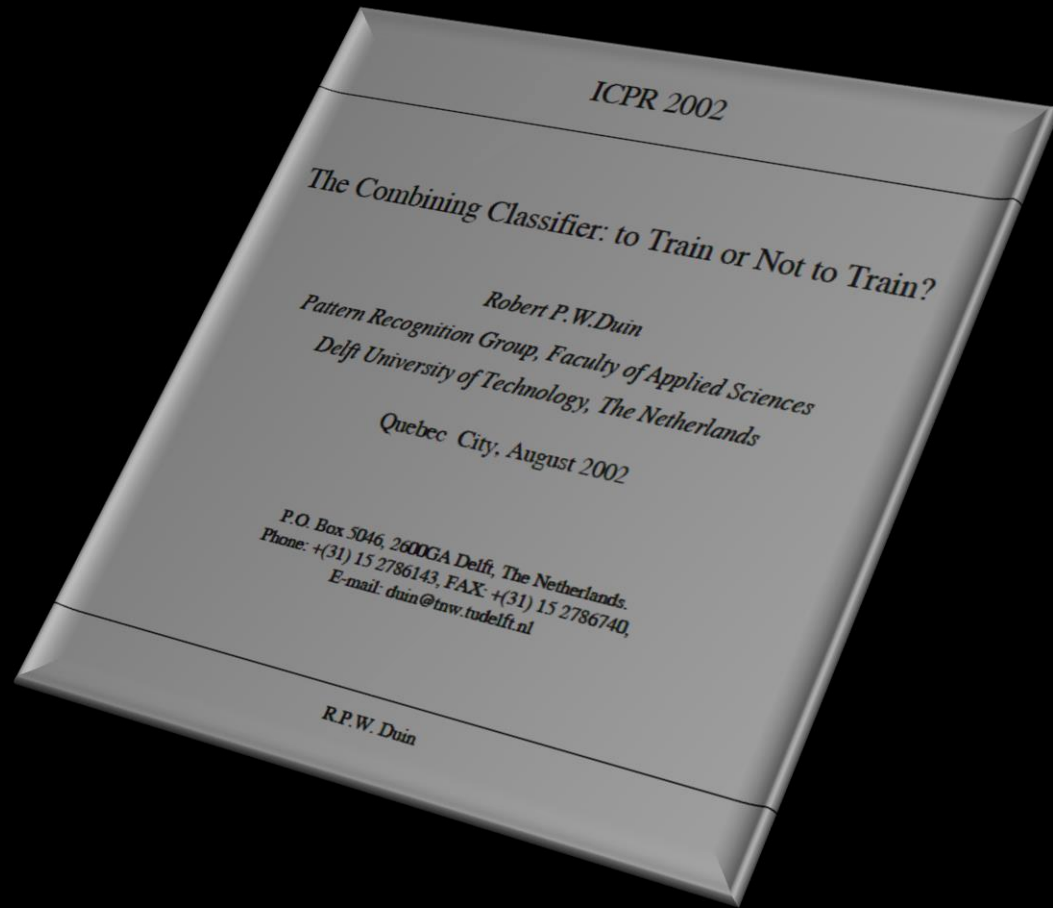
The Combining Classifier: to Train or Not to Train?



The Combining Classifier: to Train or Not to Train?



The Combining Classifier: to Train or Not to Train?



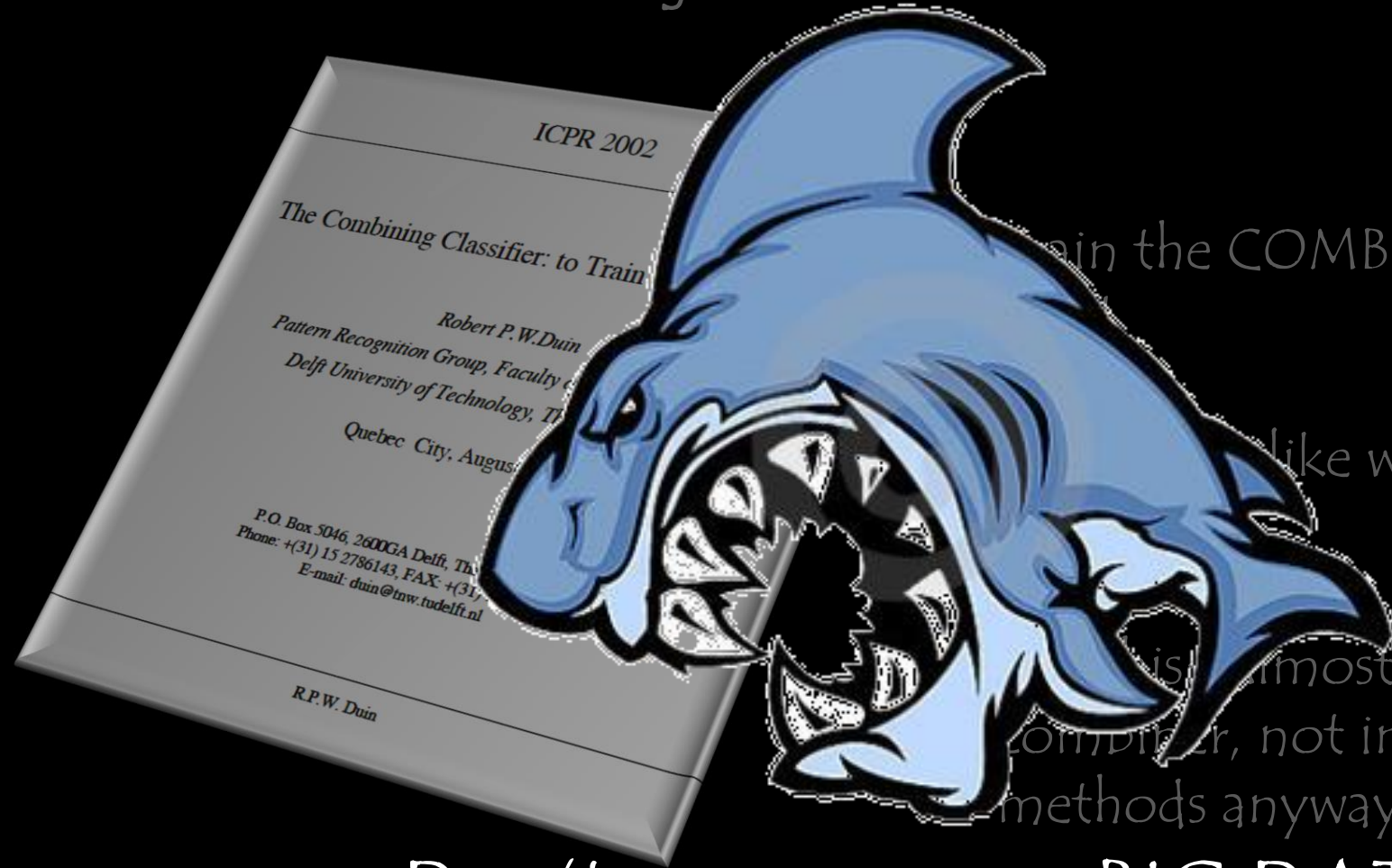
Train the COMBINER if you have "enough" data!

Otherwise, like with any classifier, we may overfit the data.

Get this: Almost NOBODY trains the combiner, not in the CLASSIC ensemble methods anyway.

Ha-ha-ha, what is "enough" data?

The Combining Classifier: to Train or Not to Train?



Train the COMBINER if you have "enough"

Like with any classifier, we may over-

is, almost NOBODY trains the
combiner, not in the CLASSIC ensemble
methods anyway.

Don't you worry... BIG DATA is coming

Ha-ha-ha, what is "enough" data?

Train the combiner and live happily ever after!

"Anchor" points

2. Diversity

All ensemble methods we have seen so far strive to keep the individual accuracy high while increasing diversity.

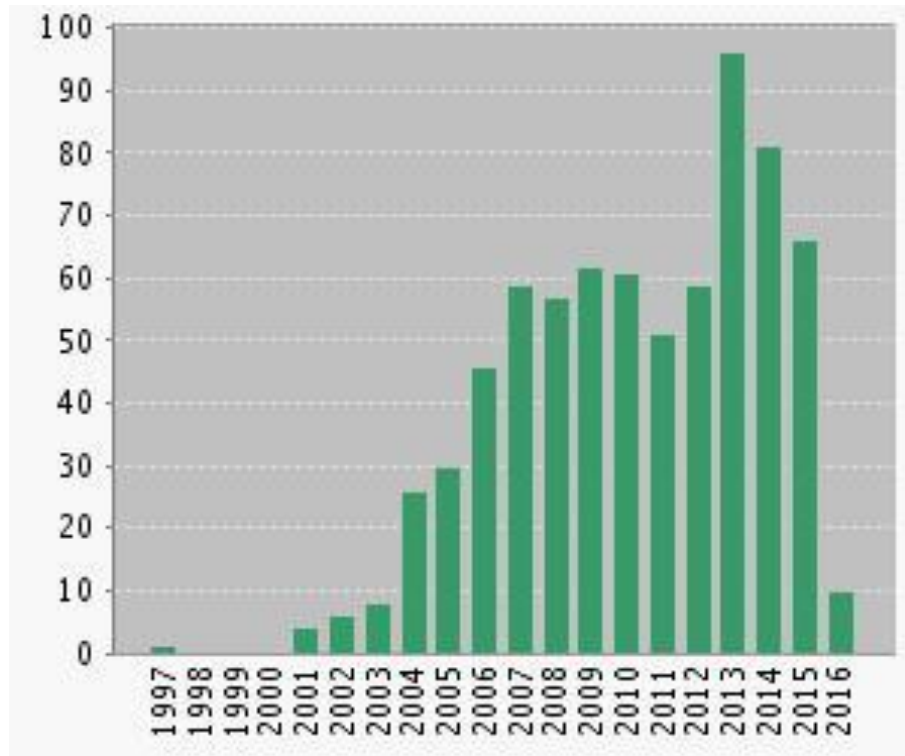
- How can we measure diversity?
- WHAT can we do with the diversity value?
 - Compare ensembles
 - Explain why a certain ensemble heuristic works and others don't
 - Construct ensemble by overproducing and selecting classifiers with high accuracy and high diversity



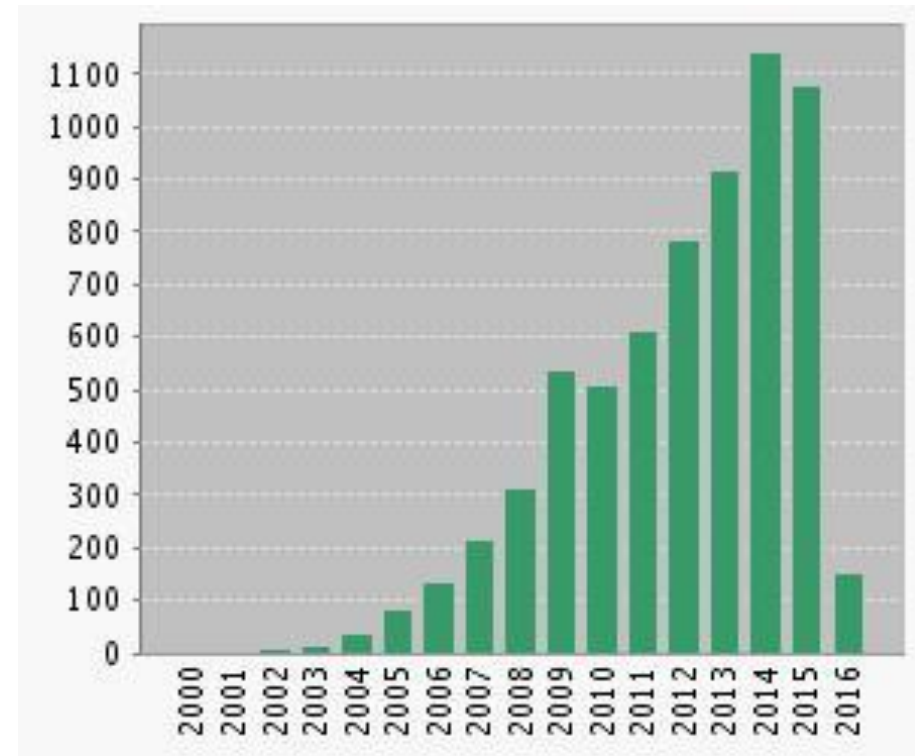
Are we still talking about diversity?

classifier + ensemble + diversity: 713 papers, 6543 citations

Published each year (713)



Cited each year (6543)



Search on 22 Feb 2016

Diversity

Measure diversity for a PAIR of classifiers

independent outputs $\neq$ independent errors
hence, use ORACLE outputs

		Classifier 2	
		correct	wrong
Classifier 1	correct	a	b
	wrong	c	d

Number of instances labelled correctly by classifier 1 and mislabelled by classifier 2

Diversity

Measure diversity for a PAIR of classifiers

		Classifier 2		Objects	C1	C2
		correct	wrong			
Classifier 1	correct	4	2	1	1	1
	wrong	1	1	2	0	0

Classifier 1

Classifier 2

Objects

1 2 3 4 5 6 7 8

C1 C2

1 1 1 1 1 1 0 1

0 0 1 1 0 1 1 0

Diversity

Measure diversity for a PAIR of classifiers

Not diverse

			Objects	C1	C2
Classifier 1	correct		1	1	1
			2	0	0
			3	1	1
			4	1	1
	wrong		5	1	1
			6	1	1
			7	0	0
			8	0	0

Classifier 2

correct

wrong

5

0

0

3

Not diverse

Diversity

Measure diversity for a PAIR of classifiers

Totally diverse

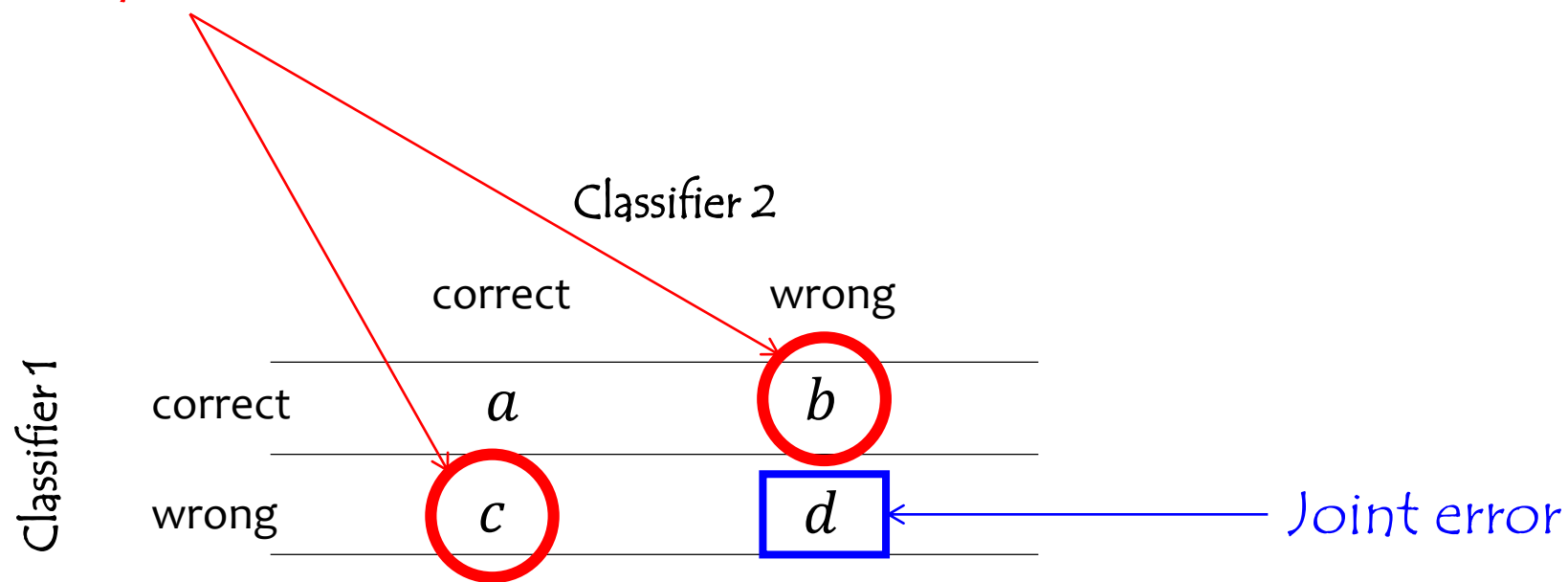
		Classifier 2	
		correct	wrong
Classifier 1	correct	0	3
	wrong	5	0

Objects	C1	C2
1	0	1
2	1	0
3	0	1
4	0	1
5	0	1
6	0	1
7	1	0
8	1	0

Diversity

Measure diversity for a PAIR of classifiers

Diversity resides here



Diversity

		Classifier 2	
		correct	wrong
Classifier 1	correct	<i>a</i>	<i>b</i>
	wrong	<i>c</i>	<i>d</i>

- Q
- kappa
- correlation (rho)
- disagreement
- double fault
- ...

A Survey of Binary Similarity and Distance Measures

Seung-Seok Choi, Sung-Hyuk Cha, Charles C. Tappert
 Department of Computer Science, Pace University
 New York, US

ABSTRACT

The binary feature vector is one of the most common representations of patterns and measuring similarity and distance measures play a critical role in many problems such as clustering, classification, etc. Ever since *Jaccard*

ecological 25 fish species [21]. Tubbs summarized seven conventional similarity measures to solve the template matching problem [28], and Zhang et al. compared those seven measures to show the recognition capability in handwriting identification [31]. Willett evaluated 13 similarity measures for binary fingerprint code [30]. Cha

Diversity

c represents the total
ad j . The total sum of
equal to n .

Table 2 [5] lists definitions of 76 binary similarity and distance measures used over the last century where S and D are similarity and distance measures, respectively.

Table 2 Definitions of Measures for binary data

$$S_{\text{JACCARD}} = \frac{a}{a+b+c} \quad (1)$$

$$S_{\text{DALE}} = \frac{2a}{2a+b+c} \quad (2)$$

$$S_{\text{CHANCE}} = \frac{2a}{2a+b+c} \quad (3)$$

$$S_{\text{SP-JACCARD}} = \frac{3a}{3a+b+c} \quad (4)$$

$$S_{\text{SHAN}} = \frac{2a}{(a+b)+(a+c)} \quad (5)$$

$$S_{\text{ROCKLANDER-1}} = \frac{a}{a+2b+2c}$$

$$S_{\text{ROCKLANDER-MICHEN}} = \frac{a+d}{a+b+c+d}$$

$$S_{\text{ROCKLANDER-2}} = \frac{2(a+d)}{2a+b+c+2d}$$

$$S_{\text{ROCKLANDER-3}} = \frac{a+d}{a+2(b+c)+d}$$

$$S_{\text{FAITH}} = \frac{a+0.5d}{a+b+c+d}$$

$$S_{\text{GOODMAN-KRUSKAL}} = \frac{a+d}{a+0.5(b+c)+d}$$

$$S_{\text{INTERSECTION}} = a$$

$$S_{\text{INTERSECTION}} = a+d$$

$$S_{\text{ROCKLANDER}} = \frac{a}{a+b+c+d}$$

$$D_{\text{HAMMING}} = b+c$$

$$D_{\text{EUCLID}} = \sqrt{b+c}$$

$$D_{\text{ROCKLANDER-EUCLID}} = \sqrt{(b+c)^2}$$

$$D_{\text{COSINE}} = (b+c)^{\frac{1}{2}}$$

$$D_{\text{MANHATTAN}} = b+c$$

$$D_{\text{CITYBLOCK}} = b+c \quad (21)$$

$$D_{\text{MINI-MAX}} = (b+c)^{\frac{1}{2}} \quad (22)$$

$$D_{\text{YAM}} = \frac{(b+c)}{4(a+b+c+d)} \quad (23)$$

$$D_{\text{DALE}} = \frac{(b+c)^2}{(a+b+c+d)^2} \quad (24)$$

$$D_{\text{CHANCE}} = \frac{n(b+c)-(b-c)^2}{(a+b+c+d)^2} \quad (25)$$

$$D_{\text{DALE}} = \frac{4bc}{(a+b+c+d)^2} \quad (26)$$

$$D_{\text{ROCKLANDER}} = \frac{b+c}{(2a+b+c)} \quad (27)$$

$$D_{\text{DALE}} = \frac{b+c}{(2a+b+c)} \quad (28)$$

$$D_{\text{DALE}} = 2 \left[1 - \frac{a}{\sqrt{a+b(a+c)}} \right] \quad (29)$$

$$D_{\text{DALE}} = 2 \left[1 - \frac{a}{\sqrt{(a+b)(a+c)}} \right] \quad (30)$$

$$D_{\text{DALE}} = \frac{a}{\sqrt{(a+b)(a+c)}} \quad (31)$$

$$S_{\text{EYRAUD}} = \frac{n^2 (na - (a+b)(a+c))}{(a+b)(a+c)(b+d)(c+d)} \quad (74)$$

$$S_{\text{TARANTULA}} = \frac{a}{(a+b)} = \frac{a(c+d)}{c(a+b)}$$

$$S_{\text{AMPLE}} = \left| \frac{\frac{a}{(a+b)}}{\frac{c}{(c+d)}} \right| = \left| \frac{a(c+d)}{c(a+b)} \right| \quad (76)$$

$$S_{\text{DALE}} = \frac{ad-bc}{\sqrt{n(a+b)(a+c)}} \quad (44)$$

$$S_{\text{DALE}} = \frac{a}{\min(a+b, a+c)} \quad (45)$$

$$S_{\text{DALE}} = \frac{a}{\max(a+b, a+c)} \quad (46)$$

$$S_{\text{DALE}} = \frac{a}{\sqrt{(a+b)(a+c)}} \quad (47)$$

$$S_{\text{DALE}} = \frac{na - (a+b)(a+c)}{n \min(a+b, a+c) - (a+b)(a+c)} \quad (48)$$

$$S_{\text{DALE}} = \frac{a}{(a+b)} + \frac{a}{(a+c)} + \frac{d}{(b+d)} + \frac{d}{(b+d)} \quad (49)$$

$$S_{\text{DALE}} = \frac{a+d}{\sqrt{(a+b)(a+c)(b+d)(c+d)}} \quad (50)$$

$$S_{\text{DALE}} = \frac{a+d}{\sqrt{(a+b)(a+c)(b+d)(c+d)}} \quad (51)$$

$$S_{\text{DALE}} = \frac{a^2}{(n+a)^2} \quad (52)$$

$$S_{\text{DALE}} = \left(\frac{\rho}{n+\rho} \right)^2 \text{ where } \rho = \frac{ad-bc}{\sqrt{(a+b)(a+c)(b+d)(c+d)}} \quad (53)$$

$$S_{\text{DALE}} = \frac{ad-bc}{\sqrt{(a+b)(a+c)(b+d)(c+d)}} \quad (54)$$

$$S_{\text{DALE}} = \frac{ad-bc}{\sqrt{(a+b)(a+c)(b+d)(c+d)}} \quad (55)$$

$$S_{\text{DALE}} = \frac{ad-bc}{\sqrt{(a+b)(a+c)(b+d)(c+d)}} \quad (56)$$

$$S_{\text{DALE}} = \frac{ad-bc}{\sqrt{(a+b)(a+c)(b+d)(c+d)}} \quad (57)$$

$$S_{\text{DALE}} = \frac{ad-bc}{\sqrt{(a+b)(a+c)(b+d)(c+d)}} \quad (58)$$

$$S_{\text{DALE}} = \frac{ad-bc}{\sqrt{(a+b)(a+c)(b+d)(c+d)}} \quad (59)$$

$$S_{\text{DALE}} = \frac{ad-bc}{\sqrt{(a+b)(a+c)(b+d)(c+d)}} \quad (60)$$

$$S_{\text{DALE}} = \frac{ad-bc}{\sqrt{(a+b)(a+c)(b+d)(c+d)}} \quad (61)$$

$$S_{\text{DALE}} = \frac{ad-bc}{\sqrt{(a+b)(a+c)(b+d)(c+d)}} \quad (62)$$

$$S_{\text{DALE}} = \frac{ad-bc}{\sqrt{(a+b)(a+c)(b+d)(c+d)}} \quad (63)$$

$$S_{\text{DALE}} = \frac{ad-bc}{\sqrt{(a+b)(a+c)(b+d)(c+d)}} \quad (64)$$

$$S_{\text{DALE}} = \frac{ad-bc}{\sqrt{(a+b)(a+c)(b+d)(c+d)}} \quad (65)$$

$$S_{\text{DALE}} = \frac{ad-bc}{\sqrt{(a+b)(a+c)(b+d)(c+d)}} \quad (66)$$

$$S_{\text{DALE}} = \frac{ad-bc}{\sqrt{(a+b)(a+c)(b+d)(c+d)}} \quad (67)$$

$$S_{\text{DALE}} = \frac{ad-bc}{\sqrt{(a+b)(a+c)(b+d)(c+d)}} \quad (68)$$

$$S_{\text{DALE}} = \frac{ad-bc}{\sqrt{(a+b)(a+c)(b+d)(c+d)}} \quad (69)$$

$$S_{\text{DALE}} = \frac{ad-bc}{\sqrt{(a+b)(a+c)(b+d)(c+d)}} \quad (70)$$

$$S_{\text{DALE}} = \frac{ad-bc}{\sqrt{(a+b)(a+c)(b+d)(c+d)}} \quad (71)$$

$$S_{\text{DALE}} = \frac{ad-bc}{\sqrt{(a+b)(a+c)(b+d)(c+d)}} \quad (72)$$

$$S_{\text{DALE}} = \frac{ad-bc}{2n} \quad (70)$$

$$S_{\text{DALE}} = \frac{ad-bc}{\sqrt{ad+a+b+c}} \quad (71)$$

$$S_{\text{DALE}} = \frac{ad-bc}{\sqrt{ad+a-(b+c)}} \quad (72)$$

$$S_{\text{DALE}} = \frac{ad-bc}{\sqrt{ad+a+b+c}} \quad (73)$$

$$S_{\text{DALE}} = \frac{ad-bc}{\sqrt{ad+a+b+c}} \quad (74)$$

$$S_{\text{DALE}} = \frac{ad-bc}{\sqrt{ad+a+b+c}} \quad (75)$$

$$S_{\text{DALE}} = \frac{ad-bc}{\sqrt{ad+a+b+c}} \quad (76)$$

$$S_{\text{DALE}} = \frac{ad-bc}{\sqrt{ad+a+b+c}} \quad (77)$$

$$S_{\text{DALE}} = \frac{ad-bc}{\sqrt{ad+a+b+c}} \quad (78)$$

$$S_{\text{DALE}} = \frac{ad-bc}{\sqrt{ad+a+b+c}} \quad (79)$$

$$S_{\text{DALE}} = \frac{ad-bc}{\sqrt{ad+a+b+c}} \quad (80)$$

$$S_{\text{DALE}} = \frac{ad-bc}{\sqrt{ad+a+b+c}} \quad (81)$$

$$S_{\text{DALE}} = \frac{ad-bc}{\sqrt{ad+a+b+c}} \quad (82)$$

$$S_{\text{DALE}} = \frac{ad-bc}{\sqrt{ad+a+b+c}} \quad (83)$$

$$S_{\text{DALE}} = \frac{ad-bc}{\sqrt{ad+a+b+c}} \quad (84)$$

$$S_{\text{DALE}} = \frac{ad-bc}{\sqrt{ad+a+b+c}} \quad (85)$$

$$S_{\text{DALE}} = \frac{ad-bc}{\sqrt{ad+a+b+c}} \quad (86)$$

$$S_{\text{DALE}} = \frac{ad-bc}{\sqrt{ad+a+b+c}} \quad (87)$$

$$S_{\text{DALE}} = \frac{ad-bc}{\sqrt{ad+a+b+c}} \quad (88)$$

$$S_{\text{DALE}} = \frac{ad-bc}{\sqrt{ad+a+b+c}} \quad (89)$$

$$S_{\text{DALE}} = \frac{ad-bc}{\sqrt{ad+a+b+c}} \quad (90)$$

$$S_{\text{DALE}} = \frac{ad-bc}{\sqrt{ad+a+b+c}} \quad (91)$$

$$S_{\text{DALE}} = \frac{ad-bc}{\sqrt{ad+a+b+c}} \quad (92)$$

$$S_{\text{DALE}} = \frac{ad-bc}{\sqrt{ad+a+b+c}} \quad (93)$$

$$S_{\text{DALE}} = \frac{ad-bc}{\sqrt{ad+a+b+c}} \quad (94)$$

$$S_{\text{DALE}} = \frac{ad-bc}{\sqrt{ad+a+b+c}} \quad (95)$$

SEVENTY SIX !!!

The inclusion or exclusion of negative matches, d in the binary similarity measures have been an ongoing issue [9, 12, 15, 16, 17, 18, 26, 27]. The Sokal & Michener, the Roger & Tanimoto, the Faith, the Ochiai II, the Cole, the Gower, Pearson I, and the Stiles etc. are included in the negative match inclusive measures. The Jaccard, the Tanimoto, the Dice & Sorenson, the Kulczynski I, the Ochiai I, the Minford, the Simpson, and the Simpson etc. are included in the negative match exclusive measures. Sokal & Michener used the negative matches do not measure any similarity between two objects. This is because in a weighted number of attributes, the matching in the objects.

In cases where the two binary states are not equally important, such as in the arguments type of binary data, the positive matches are usually more significant than the negative matches [1, 6, 10, 26]. Faith included the negative match but only gave the half credits while giving the full credits for the positive matches in eqn (10) [11]. In [4], different weights for positive and negative matches were studied. Weighted similarity measures such as weighted hamming distance or *adnno* [4] are not covered in this paper though.

Historically, all the binary measures observed above have had a meaningful performance in their respective fields. The binary similarity coefficients proposed by Peirce, Yule, and Pearson in 1900s contribute to the evolution of the various correlation based binary similarity measures. The Jaccard coefficient proposed at 1901 is still widely used in the various fields such as ecology and biology. The discussion of inclusion or exclusion of negative matches was actively arisen by Sokal & Sneath in during 1960s and by Goodman & Kruskal in 1970s. In Figure 1, the measures are arranged in historical order.

Diversity

Do we need more "NEW" pairwise diversity measures?

Looks like we don't...

And the same holds for non-pairwise measures...
Far too many already.

Take just ONE measure – kappa –  – not because it is “the best” but because one is enough.

Kappa-error diagrams

- proposed by Margineantu and Dietterich in 1997
- visualise individual accuracy and diversity in a 2-dimensional plot
- have been used to decide which ensemble members can be pruned without much harm to the overall performance

Kappa-error diagrams

		C ₂	
		correct	wrong
C ₁	correct	a	b
	wrong	c	d

error $e_1 = \frac{c+d}{a+b+c+d}$; $e_2 = \frac{b+d}{a+b+c+d}$

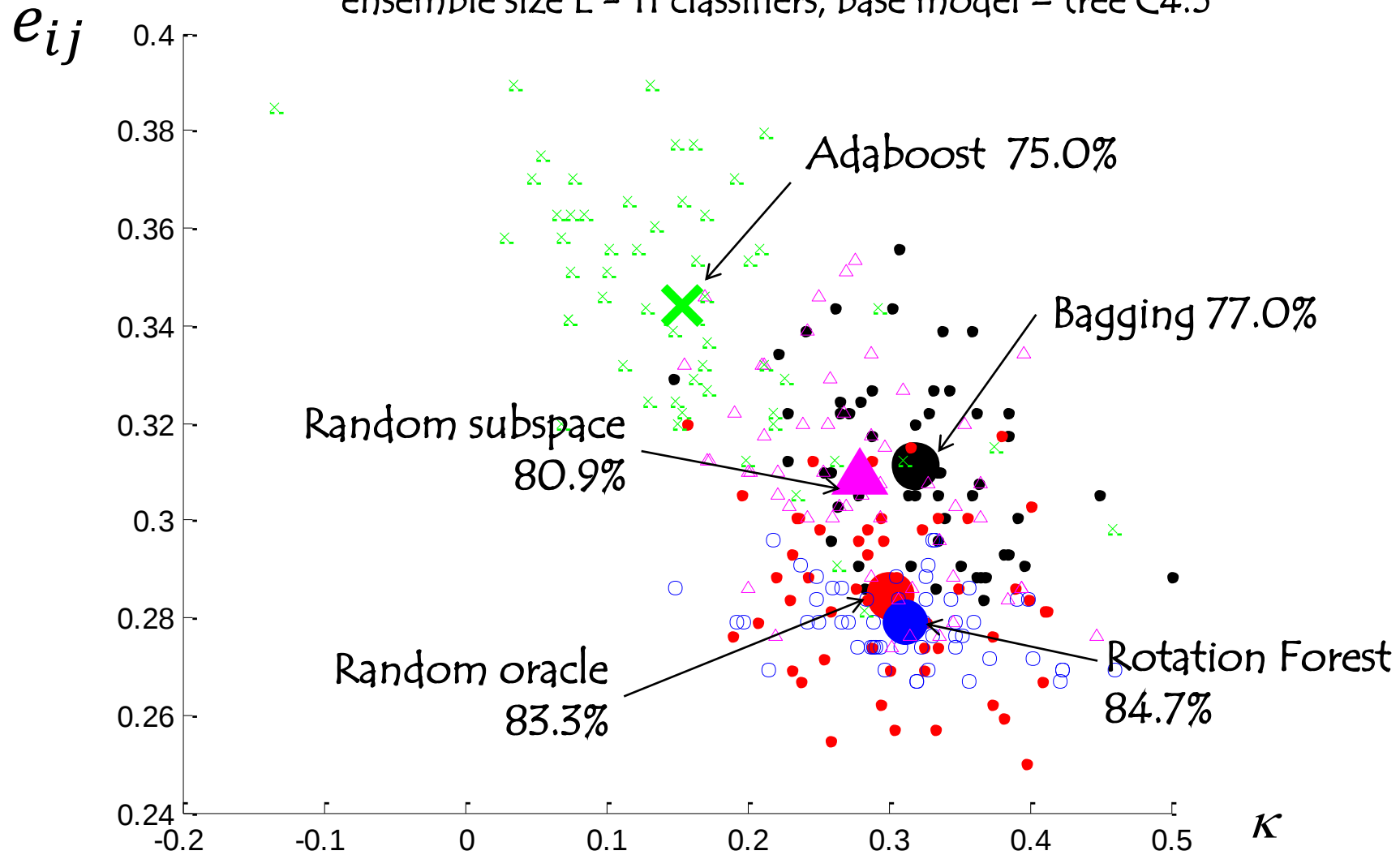
$$e = \frac{b+c+2d}{2(a+b+c+d)}$$

kappa = (observed – chance)/(1-chance)

$$K = \frac{2 (ad - bc)}{(a+b)(b+d) + (a+c)(c+d)}$$

Example

sonar data (UCI): 260 instances, 60 features, 2 classes,
ensemble size $L = 11$ classifiers, base model – tree C4.5



Kappa-error diagrams

error

$$e = \frac{b + c + 2d}{2(a + b + c + d)}$$

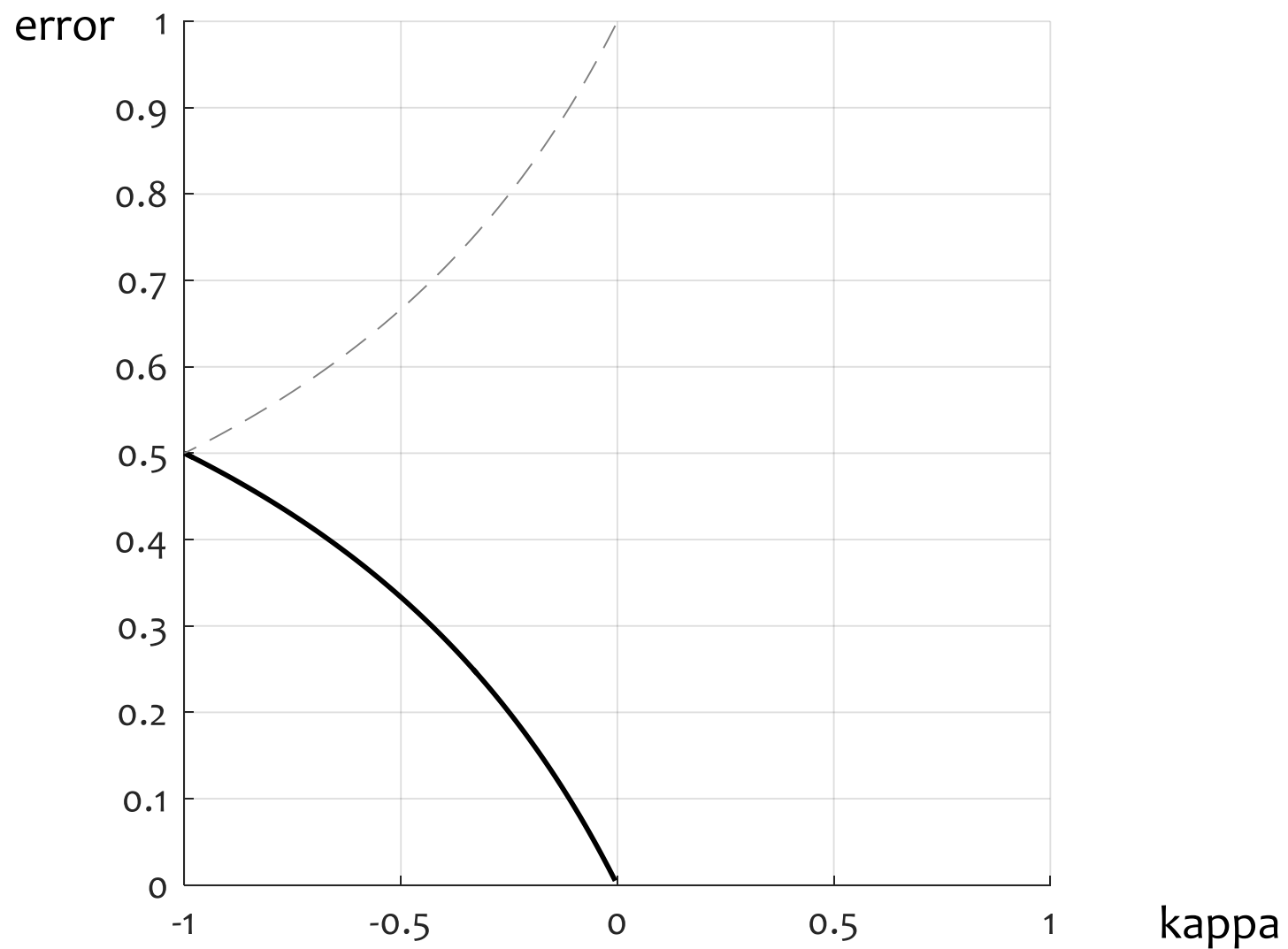
kappa

$$\kappa = \frac{2(ad - bc)}{(a + b)(b + d) + (a + c)(c + d)}$$

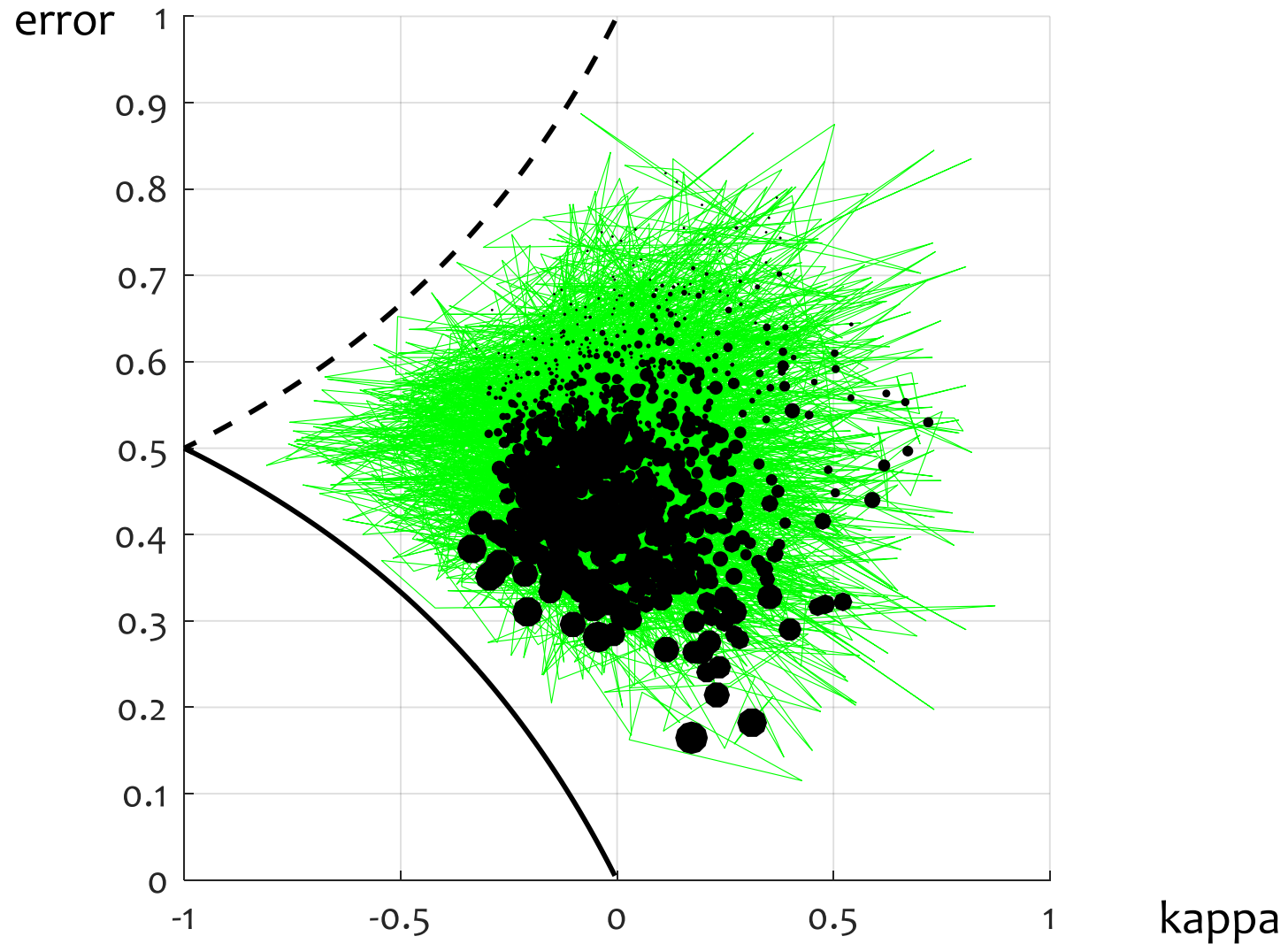
bound (tight)

$$\kappa_{\min} = \begin{cases} 1 - \frac{1}{1 - e}, & 0 < e \leq 0.5 \\ \frac{1}{1 - e}, & 0.5 < e \leq 1 \end{cases}$$

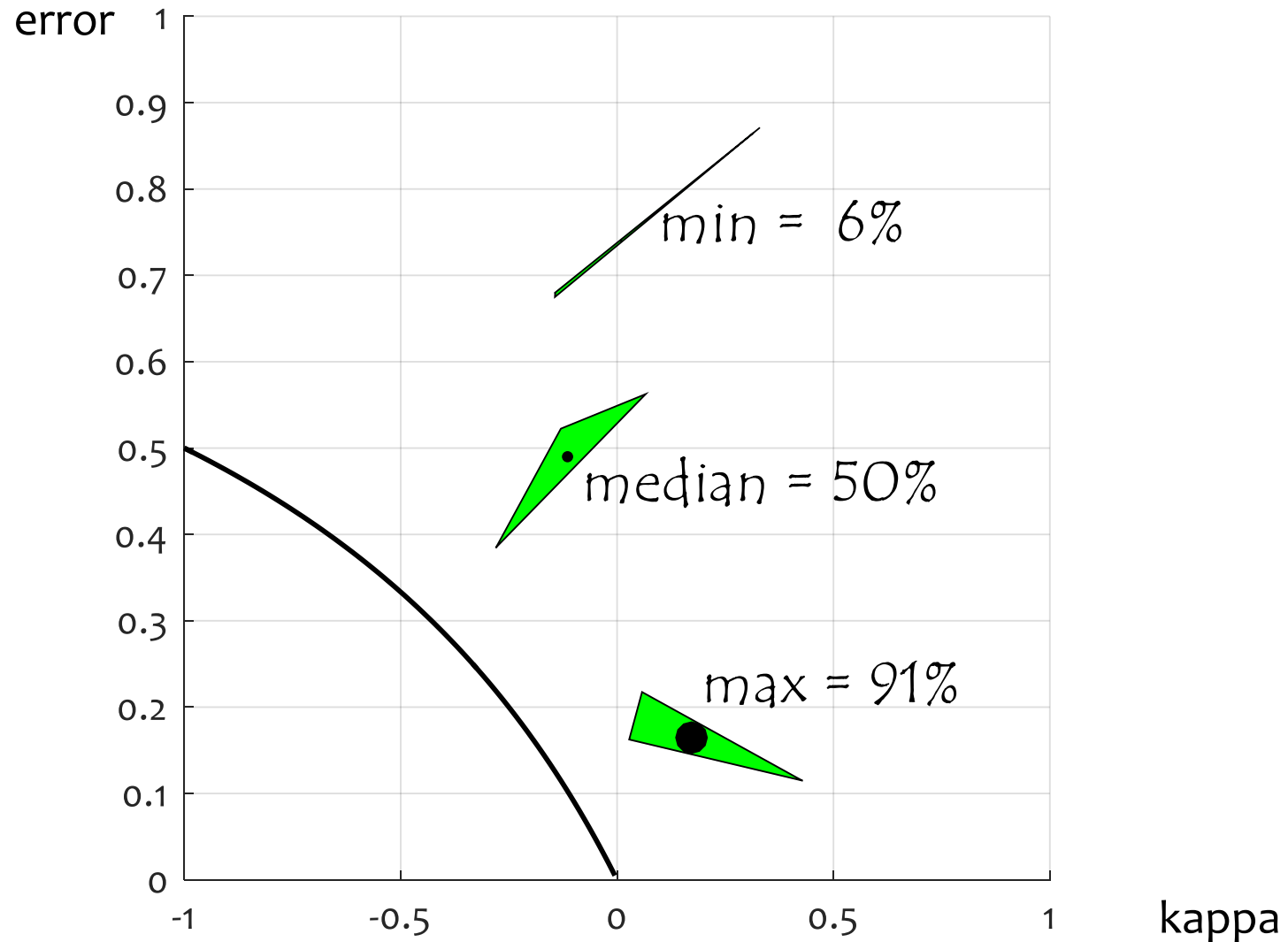
Kappa-error diagrams – bounds



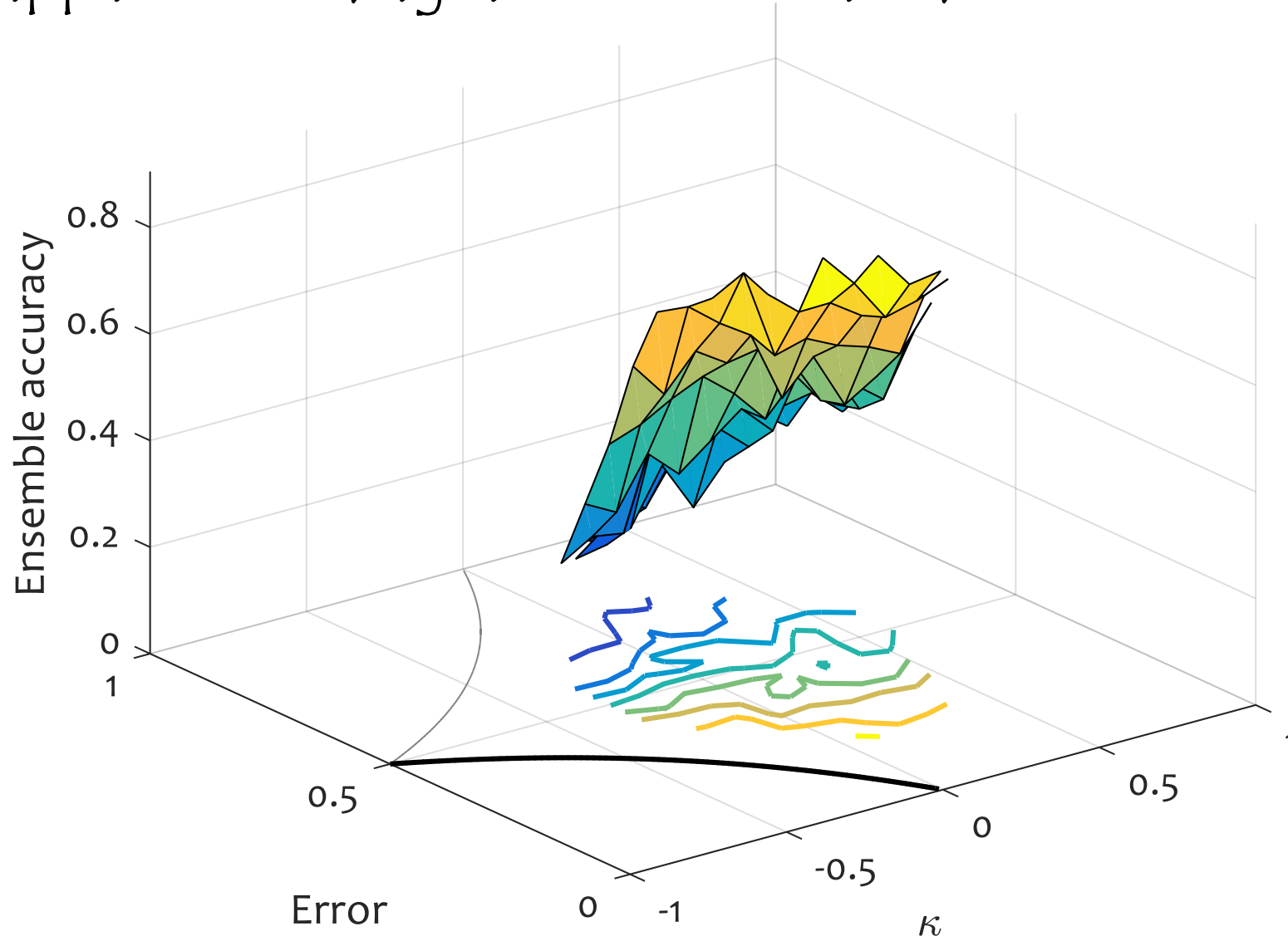
Kappa-error diagrams – simulated ensembles $L = 3$



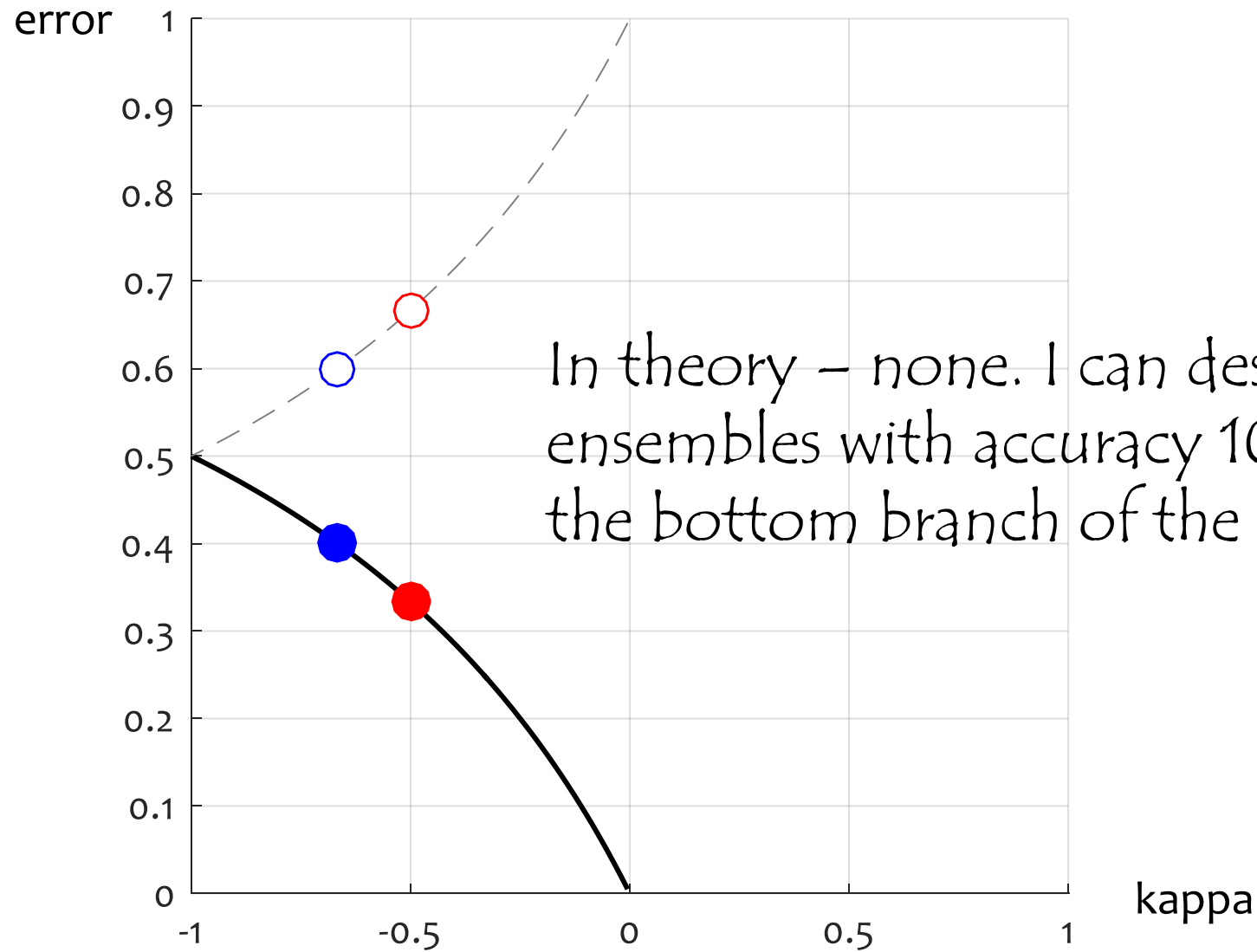
Kappa-error diagrams – simulated ensembles $L = 3$



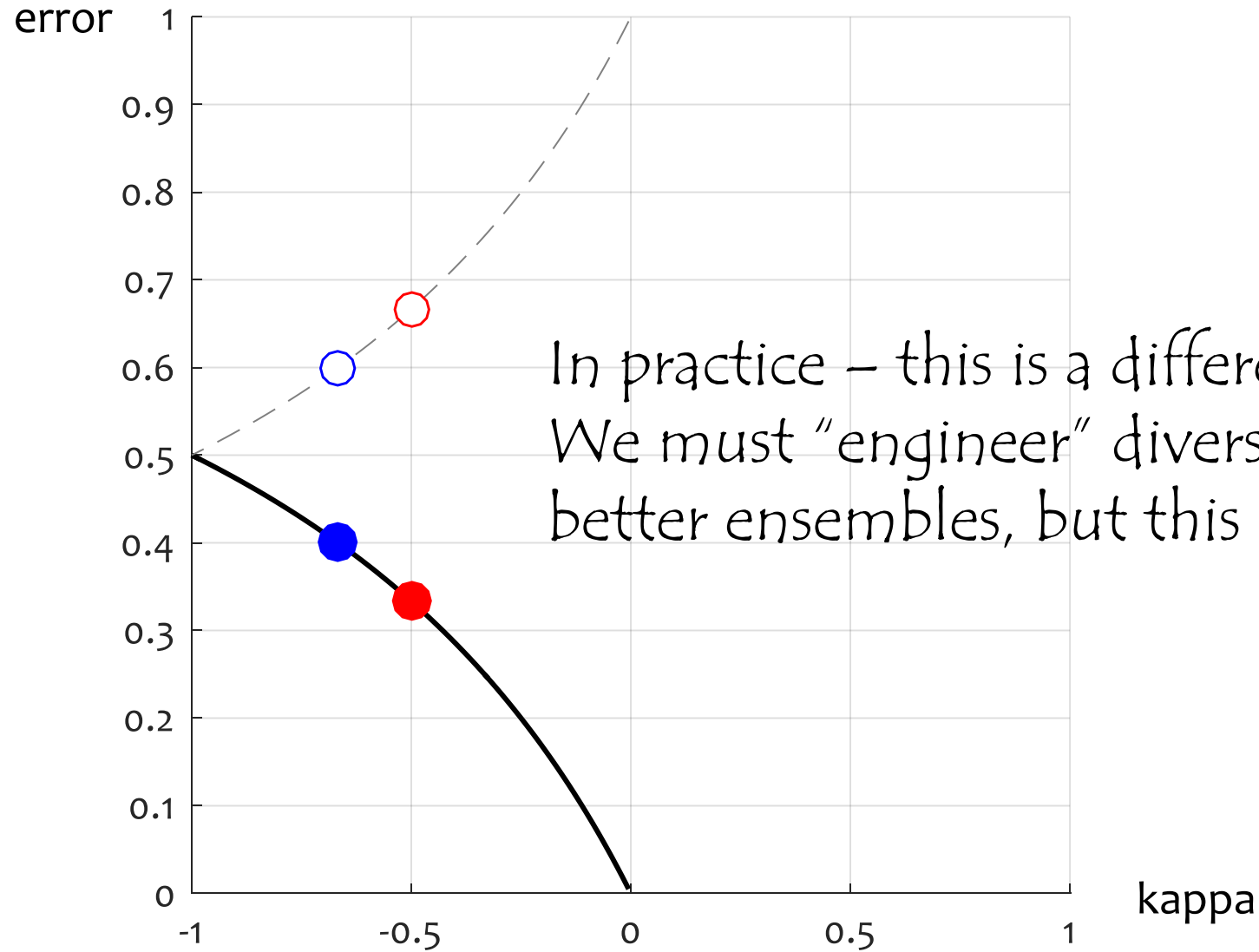
Kappa-error diagrams – simulated ensembles $L = 3$



Kappa-error diagrams – How much SPACE do we have to the bound?

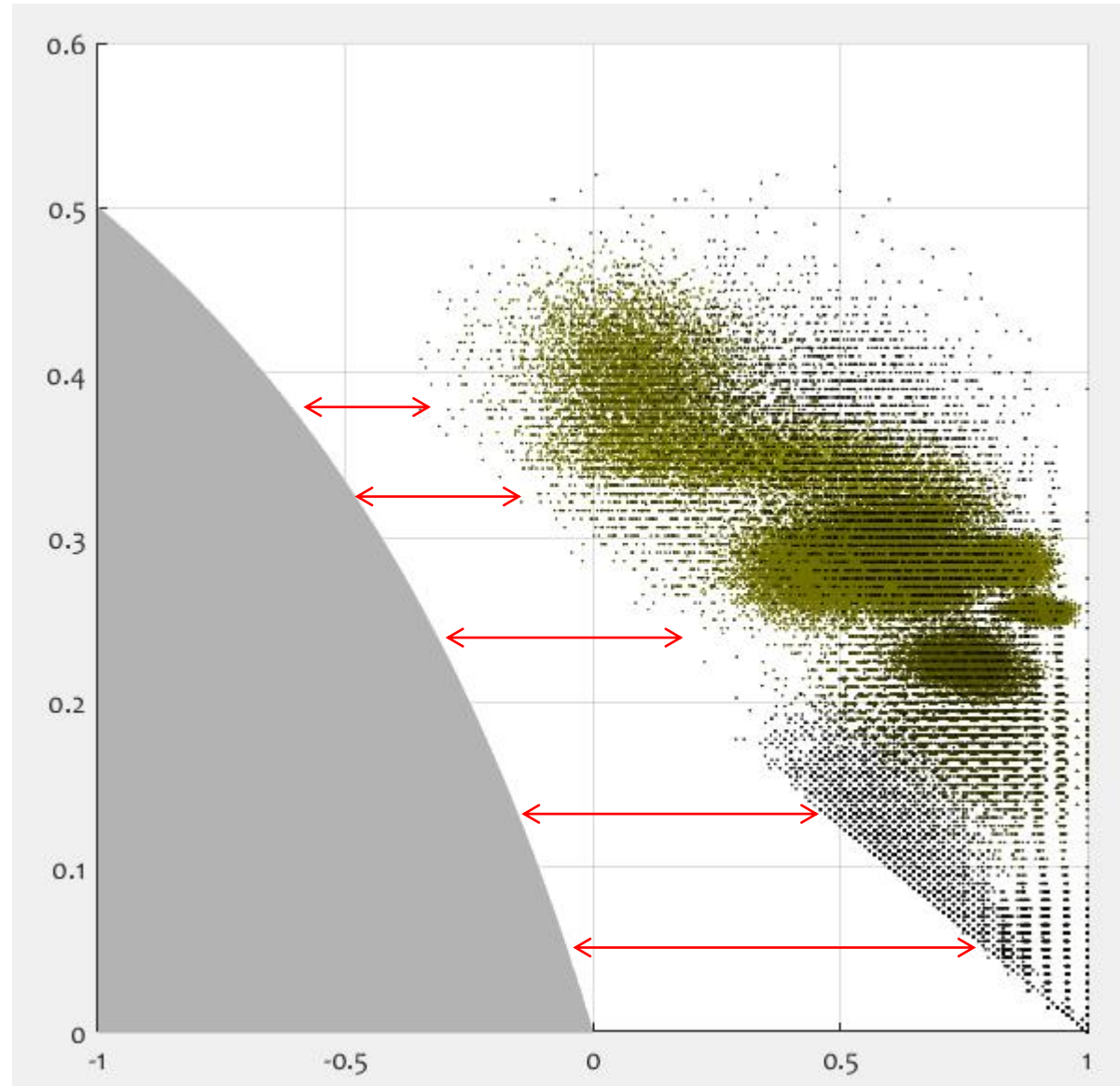


Kappa-error diagrams – How much SPACE do we have to the bound?



Kappa-error diagrams – How much SPACE do we have to the bound?

error



5 real data sets

kappa

Is there space for new classifier ensembles?

Looks like yes...

But we need revolutionary ideas about embedding diversity into the ensemble

Why is diversity so baffling?

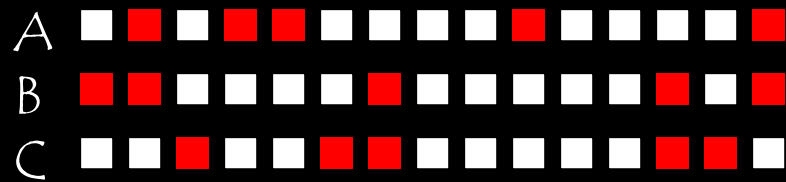
The problem is that diversity is NOT monotonically related to the ensemble accuracy.

In other words, diverse ensembles may be good or may be bad...

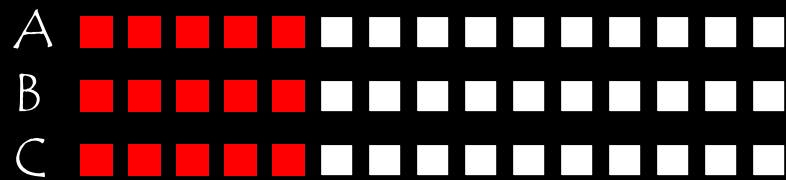
Good and Bad diversity

MAJORITY VOTE

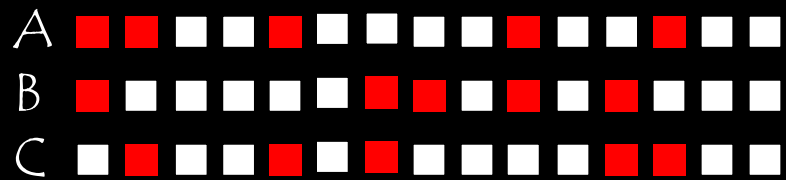
3 classifiers: A, B, C
15 objects, ■ wrong vote, ■ correct vote
individual accuracy = $10/15 = 0.667$
 P = ensemble accuracy



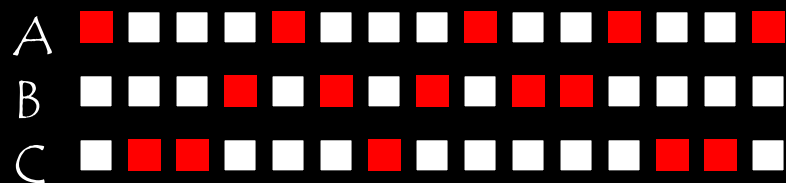
independent classifiers
 $P = 11/15 = 0.733$



identical classifiers
 $P = 10/15 = 0.667$



dependent classifiers 1
 $P = 7/15 = 0.467$

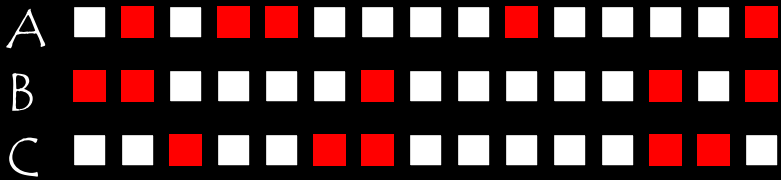


dependent classifiers 2
 $P = 15/15 = 1.000$

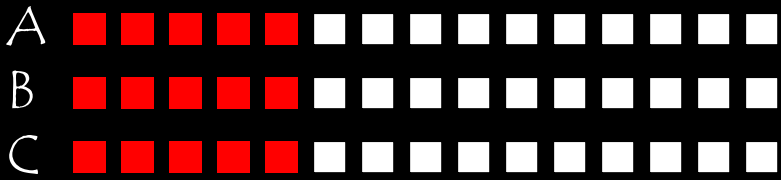
Good and Bad diversity

MAJORITY VOTE

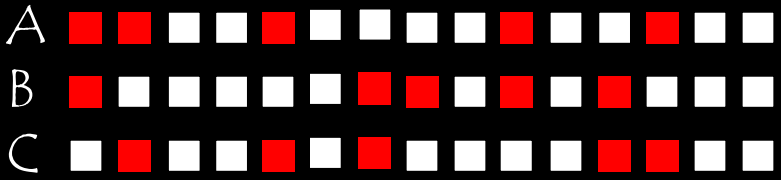
3 classifiers: A, B, C
15 objects, ■ wrong vote, ■ correct vote
individual accuracy = $10/15 = 0.667$
 P = ensemble accuracy



independent classifiers
 $P = 11/15 = 0.733$

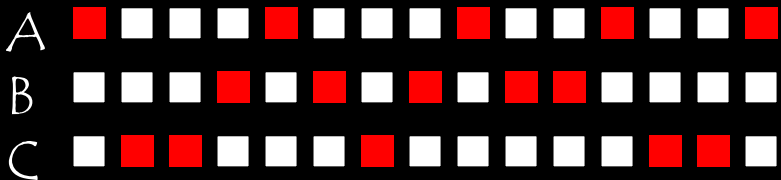


identical classifiers
 $P = 10/15 = 0.667$



dependent classifiers 1
 $P = 7/15 = 0.467$

Bad diversity



dependent classifiers 2
 $P = 15/15 = 1.000$

Good diversity

Good and Bad diversity

l_i number of classifiers with **correct** output for z_i

$L - l_i$ number of classifiers with **wrong** output for z_i

$\bar{p}$ mean individual accuracy

N number of data points

Decomposition of the Majority Vote Error

$$E_{maj} = \underbrace{(1 - \bar{p})}_{\text{Individual error}} - \underbrace{\frac{1}{NL} \sum_{\text{maj}} (L - l_i)}_{\text{Subtract GOOD diversity}} + \underbrace{\frac{1}{NL} \sum_{\text{maj}} l_i}_{\text{Add BAD diversity}}$$

The equation is annotated with three colored circles and arrows:

- A yellow circle around $(1 - \bar{p})$ with an arrow pointing to it from the text "Individual error".
- A green circle around $\frac{1}{NL} \sum_{\text{maj}} (L - l_i)$ with an arrow pointing to it from the text "Subtract GOOD diversity".
- A red circle around $\frac{1}{NL} \sum_{\text{maj}} l_i$ with an arrow pointing to it from the text "Add BAD diversity".

Good and Bad diversity

✓ ✓ ✓ ✓ ✗ ✗ ✗ This object will contribute $L - l_i = (7 - 4) = 3$ to **good** diversity

✓ ✓ ✓ ✗ ✗ ✗ ✗ This object will contribute $l_i = 3$ to **bad** diversity

Note that diversity **quantity is 3** in both cases

Ensemble Margin

The voting margin for object z_i is the proportion of <correct minus wrong votes>

$$m_i = \frac{l_i - (L - l_i)}{L}$$

✓ ✓ ✓ ✓ × × ×

$$m_i = \frac{4 - (7 - 4)}{7} = \frac{1}{7}$$

POSITIVE

✓ ✓ ✓ × × × ×

$$m_i = \frac{3 - (7 - 3)}{7} = -\frac{1}{7}$$

NEGATIVE

Ensemble Margin

Average margin

$$\bar{m} = \frac{1}{N} \sum_{i=1}^N m_i = \frac{1}{N} \sum_{i=1}^N \frac{l_i - (L - l_i)}{L}$$

Large $\bar{m}$ corresponds to BETTER ensembles...

However, nearly all diversity measures are functions of

Average absolute margin

$$|\overline{m}| = \frac{1}{N} \sum_{i=1}^N |m_i|$$

or

Average square margin

$$\overline{m^2} = \frac{1}{N} \sum_{i=1}^N m_i^2$$



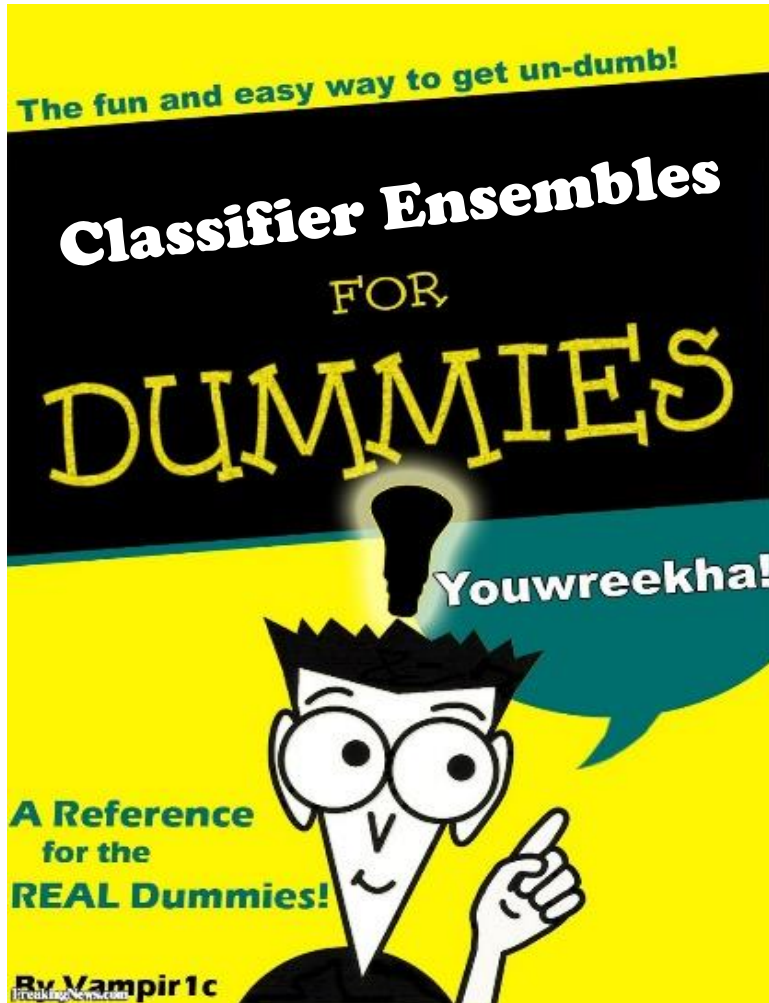
Margin has
no sign...

The bottom line is:
Diversity is not MONOTONICALLY related to
ensemble accuracy

So, stop looking for what is not there...

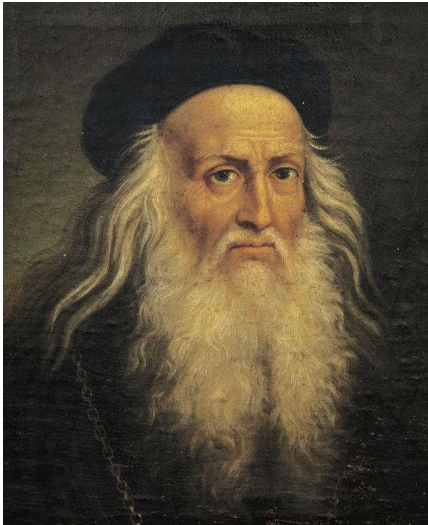
Where next in classifier ensembles?

20 years from now, what will stay in the textbooks
on classifier ensembles?



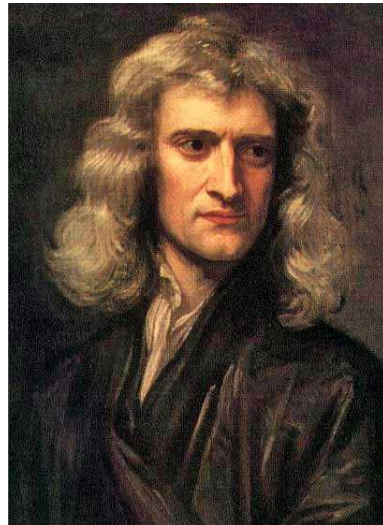
We will branch out like every other walk of science

Leonardo da Vinci
1452 - 1519



A polymath.
Invention, painting,
sculpting, architecture,
science, music, and
mathematics

Isaac Newton
1643 - 1727



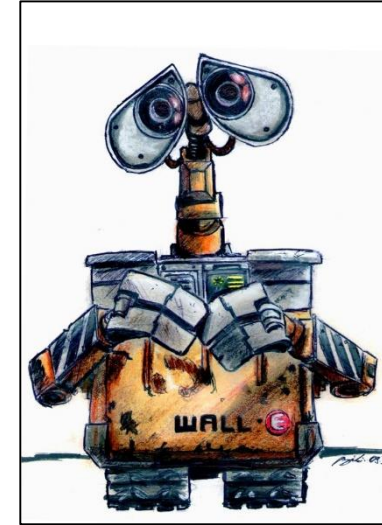
English
physicist and
mathematician

Leo Breiman
1928 - 2005



Statistician

RZ21 4X@#3
2216 - 2354



Classifier ensembler
(Concept drift,
imbalanced classes)

Ah, and Big Datist too.



A polymath is a person whose expertise spans a significant number of different subject areas.

Instead of conclusions :)

For the winner



by my favourite illustrator Marcello Barenghi

Well, I'll give you a less crinkled one :)

A guessing game:
CITATIONS
on *Web of Science*
23/02/2016

"Deep learning"
1,490

"Big data"
8,151

Sort 2-9

from highest to lowest

1. "Classifier Ensemble*" 788

2. (1) + "concept drift"

3. (1) + "rotation forest"

4. (1) + adaboost

5. (1) + (imbalanced or unbalanced)

6. (1) + diversity

7. (1) + combiner

8. (1) + "big data"

9. (1) + "deep learning"

Citation counts

"Deep learning"
1,490

"Big data"
8,151

1. "Classifier Ensemble*"	788
2. (1) + "concept drift"	18
3. (1) + "rotation forest"	27
4. (1) + adaboost	78
5. (1) + (imbalanced or unbalanced)	37
6. (1) + diversity	205
7. (1) + combiner	11
8. (1) + "big data"	1
9. (1) + "deep learning"	0

Solution

"Classifier Ensemble*"
 788

"Deep learning"
 1,490

"Big data"
 8,151

6. (1) + diversity	205
4. (1) + adaboost	78
5. (1) + (imbalanced or unbalanced)	37
3. (1) + "rotation forest"	27
2. (1) + "concept drift"	18
7. (1) + combiner	11
8. (1) + "big data"	1
9. (1) + "deep learning"	0

And thank you for listening to me!



The Leverhulme Trust

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